Title: Weak solutions to the Navier–Stokes inequality with arbitrary energy profiles.

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Abstract: In the talk we will focus on certain constructions of weak solutions to the Navier–Stokes inequality,

\[ u \cdot (u_t - \nu \Delta + (u \cdot \nabla)u + \nabla p) \leq 0 \]

on \( \mathbb{R}^3 \). Such vector fields satisfy both the strong energy inequality and the local energy inequality (but not necessarily solve the Navier–Stokes equations). Given \( T > 0 \) and a nonincreasing energy profile \( e: [0, T] \to [0, \infty) \) we will construct a weak solution to the Navier–Stokes inequality that is localised in space and whose energy profile \( \|u(t)\|_{L^2(\mathbb{R}^3)} \) stays arbitrarily close to \( e(t) \) for all \( t \in [0, T] \).

The relevance of such solutions is that, despite not satisfying the Navier–Stokes equations, they do satisfy the partial regularity theory of Caffarelli, Kohn & Nirenberg (Comm. Pure Appl. Math., 1982). In fact, Scheffer’s constructions of weak solutions to the Navier–Stokes inequality with blow-ups (Comm. Math. Phys., 1985 & 1987) show that the Caffarelli, Kohn & Nirenberg’s theory is sharp for such solutions. Namely, his construction admits a finite-time blow-up on a Cantor set whose Hausdorff dimension is greater than \( \xi \), for any preassigned \( \xi \in (0, 1) \). We will discuss the main ideas of his constructions.

Moreover, we will show how our approach can be used to obtain a stronger result than Scheffer’s. Namely, we obtain weak solutions to the Navier–Stokes inequality with both blow-up and a prescribed energy profile.

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