My research is in a branch of pure mathematics called topology, more specifically algebraic topology and string topology. I am also interested in the intersection of mathematics, education, and art. This document outlines my work, interests, and future research in both math and interdisciplinary areas.

1 Mathematics

1.1 Background

String topology is the study of the algebraic structure on homology of the free loop space of a manifold $M$. This structure was called the loop product on $H_*(LM)$, described by Chas and Sullivan [CS] in 1999, arising from the intersection of families of loops and concatenating loops.

Chas and Sullivan generalized a Lie algebra structure that Goldman described in the eighties [Go] on the vector space of homotopy classes of free loops on orientable surfaces, possibly with boundary. This Lie bracket, called the Goldman bracket, was described on homotopy classes of loops. The Goldman bracket of two loops, $\alpha$ and $\beta$ on a surface is given by taking the formal sum of loops that are created by concatenation at each of the intersection points of $\alpha$ and $\beta$, the order and direction of the new concatenated loop is determined by the orientation of the loops compared to the orientation of the manifold.

It is natural that the Goldman bracket, since it is defined with loop intersections, gives information about the structure of intersections.

Theorem 1.1. (Goldman) [Go] Let $\alpha$ be a free homotopy class of a loop that has a simple (no self-intersection) representative, and let $\beta$ be another homotopy class of a loop. If $[\alpha, \beta] = 0$, then $\alpha$ and $\beta$ have disjoint representatives.

Later, Chas generalized this result and made some discoveries with the Goldman bracket with minimal intersection numbers. She found that if the bracket was taken of two classes of loops, one of which is simple, then the number of terms in the Goldman bracket is the minimal intersection number of the two curves. Gadgil's work shows
that the Goldman bracket characterizes homeomorphisms of surfaces with boundary. There are still a lot of open problems in this area.

In Chas and Sullivan’s foundational paper ”String Topology,” they highlight the various algebraic structures arising from the free loop space of a manifold. Their inspiration was from the Goldman bracket, which Chas and Sullivan’s higher dimensional analogue they called the string bracket. The string bracket gives the equivariant homology of the free loop space of an n-manifold, $H_*^{S^1}(LM)$, also called string homology, a structure of a graded, infinite-dimensional Lie algebra, as follows:

$$[\cdot, \cdot] : H_i^{S^1}(LM) \otimes H_j^{S^1}(LM) \to H_{i+j+2-n}^{S^1}(LM).$$

This bracket is given by intersecting two families of unbased loops and concatenating to get a new family of loops. The string bracket on $H_0^{S^1}(LM)$ corresponds to the Goldman bracket.

### 1.2 Current and Future Work

Most of my research work is an attempt to answer the following questions:

**Question 1:** What is the Lie algebra structure on String Homology?

**Question 2:** What can the algebraic structures from string topology say about geometric information of a manifold.

I am interested in the Lie algebra structure on string homology. Westerland computed string homology of spheres and projective spaces over $\mathbb{Z}_2$ coefficients [We], and Basu, in his Ph.D. thesis [Ba], computed the string bracket and string homology of spheres with rational coefficients. I have made computations of string homology and the string bracket over the integers for certain manifolds, such as spheres, projective spaces, and surfaces [?]. Basu found the bracket of spheres to be basically trivial, but the bracket of products of spheres was highly non-trivial. Basu commented that the bracket may possibly be used to detect products. In my computations of string homology with integral coefficients, a lot of interesting torsion comes up. I am also interested in how this new information about torsion can answer geometric questions about manifolds, in particular when it comes to characterisations about intersections of loops.

In the non-equivariant case, $H_*(LM)$ also has a Lie bracket structure called the loop
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bracket. In a paper by Gatsinzi and Kwashira [GK], a sum of positive Witt algebras was discovered in the loop bracket Lie algebra structure of $H_*(LM)$. Basu’s thesis briefly mentioned a Witt subalgebra in the string homology of products of odd spheres. I am working on finding more Witt structures in string homology.

Another project I am working on is computing the higher string bracket operations mentioned in the end of [CS]. String homology also carries the structure of a Lie-$\infty$ algebra. It is currently not yet proven if this is a homotopy invariant, but it seems likely that it is.

I am also interested in the low-dimensional aspect of String Topology by working on some conjectures and problems in regards to the Goldman Lie algebra. The following conjecture, which has now been proven by Kabiraj [Ka] was posed by Chas:

**Theorem 1.2.** The center of the Goldman Lie algebra on a surface with boundary is generated by loops which are homotopic to the boundary or the contractible loop.

Kabiraj proved this conjecture using hyperbolic geometry. Etingof proved that the center of the Goldman Lie algebra for surfaces was infinite cyclic, generated by the class of the constant loop [E]. I have been working on an alternate proof of the above theorem using combinatorial group theoretic tools, following Chas’ papers [Cha1] [Cha2].

**Question:** Can an extension of this statement be made for the string Lie algebra? My goal is to try to get a complete characterization of the center of the string Lie algebra.

I am also interested in answering the question about whether or not the Goldman Lie algebra is finitely generated. I have shown that the Goldman Lie algebra for the closed torus is finitely generated, by three generators, as a Lie algebra over $\mathbb{Q}$, and it is not finitely generated over $\mathbb{Z}$ [?]. These results correspond to the results given by Kawazumi, Kuno, and Toda [KKT], where they showed that the homological Goldman Lie algebra is finitely generated. Their results were about the homological Goldman Lie algebra, a variant that is defined on the first homology group of the surface and the intersection form. There is actually a surjective Lie algebra homomorphism from the Goldman algebra to the homological Goldman lie algebra.

**Proposed Research:** Show that for oriented surfaces with boundary, the Goldman
Lie Algebra is not finitely generated.

My goal is to determine results from the Goldman Lie algebra, and in the spirit of Chas and Sullivan’s work, make generalizations with the String homology Lie algebra.

Mathematics Publications:


2 Interdisciplinary Work

I have always been interested in the visual arts, and recently I have been incorporating this into how I teach mathematics.

2.1 Mathematics, Art, and Education

I am currently working on writing a paper about a project I gave my linear algebra students about linear transformations and the portraits of the artist Amedeo Modigliani, who was known for making portraits of actual people where he incorporated elongation of their faces, necks, or body. The project is about whether or not Modigliani’s portraits can be characterized with a linear transformation from the photo of Modigliani’s subjects to Modigliani’s portrait of the same subject. The purpose of the project is for the students to apply what they learned about the geometric properties of a linear transformation, and see if they understand that in a real-life context of the transformation of an image. It also gives them a hands-on experience with computing, writing a program, or using other technology tools to research a problem. This project is a work in progress that I plan on submitting as a suggested project that blends math, art, and education.
2.2 Mathematics and Art

Lately, I have been using mathematics as an inspiration to create works of art. I have a series of block prints I made with inspiration coming from algebraic topology and knot theory, and also Chinese arts and crafts. I wrote a paper, [4], on the process of making these prints and the mathematical inspiration for them.

I also just started new research on mon, which are Japanese family crests. Mon are very simple, black and white designs that are very geometric with a lot of symmetry. The designs seem to be constructed with only a ruler and straightedge, which is something I am still investigating. I plan on writing a paper on the history of mon, the construction and drafting of mon, their symmetries, and possible use as a teaching activity.

Interdisciplinary Publications:


References


[He] Richard Hepworth, *String Topology for Lie Groups*


[Ku] Alexander Kupers, *An elementary proof of the string topology structure of compact oriented surfaces*


[We] Craig Westerland, *String Homology of Spheres and Projective Spaces*