Double Machine Learning for Causal and Treatment Effects

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This presentation is based on the following papers:

  with Alexandre Belloni, I. Fernandez-Val, Christian Hansen

• "Double Machine Learning for Causal and Treatment Effects"
  ArXiv 2016, with Denis Chetverikov, Esther Duflo, Christian Hansen, Mert Demirer, Whitney Newey
Main goal: Provide general framework for estimating and doing inference about a low-dimensional parameter ($\theta_0$) in the presence of high-dimensional nuisance parameter ($\eta_0$) which may be estimated with the new generation of nonparametric statistical methods, branded as “machine learning” (ML) methods, such as

- random forests,
- boosted trees,
- lasso,
- ridge,
- deep and standard neural nets,
- gradient boosting,
- their aggregations,
- and cross-hybrids.
• Lots of recent work on inference based on lasso-type methods
  • e.g. Belloni, Chen, Chernozhukov, and Hansen (2012); Belloni, Chernozhukov, Fernández-Val, and Hansen (2015); Belloni, Chernozhukov, and Hansen (2010, 2014); Belloni, Chernozhukov, Hansen, and Kozbur (2015); Belloni, Chernozhukov, and Kato (2013a, 2013b); Belloni, Chernozhukov, and Wei (2013); Farrell (2015); Javanmard and Montanari (2014); Kozbur (2015); van de Geer, Bühulmann, Ritov, and Dezeure (2014); Zhang and Zhang (2014)

• Relatively little work on other ML methods in high-dimensional setting, with exceptions, e.g., Chernozhukov, Hansen, and Spindler (2015), Athey and Wager (2015);
  • Will build on the general framework in Chernozhukov, Hansen, and Spindler (2015)
Two main points:

I. The ML methods seem remarkably effective in prediction contexts. However, good performance in prediction does not necessarily translate into good performance for estimation or inference about “causal” parameters. In fact, the performance can be poor.
Two main points:

I. The ML methods seem remarkably effective in prediction contexts. However, good performance in prediction does not necessarily translate into good performance for estimation or inference about “causal” parameters. In fact, the performance can be poor.

II. By doing "double" ML or “orthogonalized” ML, and sample splitting, we can construct high quality point and interval estimates of "causal" parameters.
Main Points via a Partially Linear Model

Illustrate the two main points in a canonical example:

\[ Y = D\theta_0 + g_0(Z) + U, \quad E[U \mid Z, D] = 0, \]

- \( Y \) - outcome variable
- \( D \) - policy/treatment variable
  - \( \theta_0 \) is the target parameter of interest
- \( Z \) is a high-dimensional vector of other covariates, called “controls” or “confounders”

\( Z \) are confounders in the sense that

\[ D = m_0(Z) + V, \quad E[V \mid Z] = 0 \]

where \( m_0 \neq 0 \), as is typically the case in observational studies.
Point I. “Naive” or Prediction-Based ML Approach is Bad

- Predict $Y$ using $D$ and $Z$ – and obtain

$$D\hat{\theta}_0 + \hat{g}_0(Z)$$

- For example, estimate by alternating minimization– given initial guesses, run Random Forest of $Y - D\hat{\theta}_0$ on $Z$ to fit $\hat{g}_0(Z)$ and the Ordinary Least Squares on $Y - \hat{g}_0(Z)$ on $D$ to fit $\hat{\theta}_0$; Repeat until convergence.

- Excellent prediction performance! BUT the distribution of $\hat{\theta}_0 - \theta_0$ looks like this:

![Histogram and normal distribution comparison](image.png)
Point II. The “Double” ML Approach is Good

1. Predict \( Y \) and \( D \) using \( Z \) by

\[
\hat{E}[Y\mid Z] \quad \text{and} \quad \hat{E}[D\mid Z],
\]

obtained using the Random Forest or other "best performing ML" tools.

2. Residualize \( \hat{W} = Y - \hat{E}[Y\mid Z] \) and \( \hat{V} = D - \hat{E}[D\mid Z] \)

3. Regress \( \hat{W} \) on \( \hat{V} \) to get \( \hat{\theta}_0 \).

- Frisch-Waugh-Lovell (1930s) style. The distribution of \( \hat{\theta}_0 - \theta_0 \) looks like this:
Moment conditions

The two strategies rely on very different moment conditions for identifying and estimating $\theta_0$:

$$E[\psi(W, \theta_0, \eta_0)] = 0$$

$$\psi_1(W, \theta_0, \eta) = (Y - D\theta_0 - g_0(Z))D \quad (1)$$

$$\psi_1(W, \theta_0, \eta_0) = E[((Y - E[Y|Z]) - (D - E[D|Z])\theta_0)(D - E[D|Z])] \quad (2)$$

- (1) - Regression adjustment score, with
  $$\eta = g(Z), \quad \eta_0 = g_0(Z),$$
- (2) - Neyman-orthogonal score (Frisch-Waugh-Lovell), with
  $$\eta = (\ell(Z), m(Z)), \quad \eta_0 = (\ell_0(Z), m_0(Z)) = (E[Y | Z], E(D | Z))$$

Both estimators solve the empirical analog of the moment conditions:

$$E_n[\psi(W, \theta, \hat{\eta}_0)] = 0$$

where instead of unknown nuisance functions we plug-in their ML-based estimators, obtained using auxiliary (set-aside) sample.
Key Difference between (1) and (2) is Neyman Orthogonality

- The Neyman orthogonality condition:
  \[
  D = \partial_\eta E\psi(W, \theta_0, \eta)\big|_{\eta=\eta_0} = 0
  \]

  Heuristically, the Neyman orthogonality conditions says that the moment condition remains "valid" under "local" mistakes in the nuisance function.

- The condition \textit{does hold} for the score (2) and \textit{fails to hold} for the score (1),
Heuristics: The Role of Neyman Orthogonality

- We have expansion

\[ J \sqrt{n}(\hat{\theta} - \theta_0) = A_n + \sqrt{n}D(\hat{\eta} - \eta_0) + C\sqrt{n}O(\|\hat{\eta} - \eta_0\|^2) + o_p(1), \]

where the leading term is well-behaved and approximately Gaussian under weak conditions, if sample-splitting is used and \( \|\hat{\eta} - \eta_0\| \to 0. \)

- When \( D \neq 0, \) since \( \|\hat{\eta} - \eta_0\| = O_P(n^{-\varphi}), 0 < \varphi < 1/2, \)

\[ \sqrt{n}D(\hat{\eta} - \eta_0) \text{ is of order } \sqrt{nn^{-\varphi}} \to \infty. \]

and the estimator without Neyman orthogonality is not root-n consistent.
Heuristics: The Role of Neyman Orthogonality?

- Under Neyman orthogonality $D = 0$, then
  \[ \sqrt{n}D(\hat{\eta} - \eta) = 0, \]
  and for root-n consistency we only need,
  \[ C \sqrt{n}O(\|\hat{\eta} - \eta_0\|^2) \rightarrow 0, \]
  which requires $\|\hat{\eta} - \eta_0\| = o_P(n^{-1/4})$ if $C \gg 0$.
- This is attainable rate for many ML estimators, especially aggregated estimators.
- In some problems $C = 0$, like optimal IV problem in Belloni et al (2010) or when $m_0 = 0$ (as in the randomized control trials).
Heuristics: The Role of Sample Splitting

• Need to show

\[ A_n = \mathbb{G}_n(\psi(W, \theta_0, \hat{\eta})) \sim N(0, \Omega), \]

where \( \mathbb{G}_n \) is the empirical process.

• So we need

\[ \mathbb{G}_n(\psi(W, \theta_0, \hat{\eta}) - \mathbb{G}_n(\psi(W, \theta_0, \eta_0) \to P 0. \]

• If \( \hat{\eta} \) is based on the auxiliary sample, not used in the main estimation, then this follows from \( \|\hat{\eta} - \eta_0\| \to 0 \) and Chebyshev inequality.

• If \( \hat{\eta} \) is based on the main sample, need maximal inequalities to control

\[ \sup_{\eta \in \mathcal{M}_n} \left| \mathbb{G}_n(\psi(W, \theta_0, \eta) - \mathbb{G}_n(\psi(W, \theta_0, \eta_0) \right| \]

We need to control the rate of entropy growth for \( \mathcal{M}_n \ni \hat{\eta} \ldots \]

• Of course, we did this, see our "Program Evaluation Paper.." in Econometrica... The condition is reasonable, but it might be hard to check for each new ML method...
General Results for Moment Condition Models

Moment conditions model:

$$E[\psi_j(W, \theta_0, \eta_0)] = 0, \quad j = 1, \ldots, d_\theta$$  \hspace{1cm} (3)

- $\psi = (\psi_1, \ldots, \psi_{d_\theta})'$ is a vector of known score functions
- $W$ is a random element; observe random sample $(W_i)_{i=1}^N$ from the distribution of $W$
- $\theta_0$ is the low-dimensional parameter of interest
- $\eta_0$ is the true value of the nuisance parameter $\eta \in T$ for some convex set $T$ equipped with a norm $\| \cdot \|_e$ (can be a function or vector of functions)
Key Ingredient I: Neyman Orthogonality Condition

Key orthogonality condition:

\( \psi = (\psi_1, \ldots, \psi_{d_\theta})' \) obeys the orthogonality condition with respect to \( T \subset T \) if the Gateaux derivative map

\[
D_{r,j}[\eta - \eta_0] := \partial_r \left\{ E_P \left[ \psi_j(W, \theta_0, \eta_0 + r(\eta - \eta_0)) \right] \right\}
\]

- exists for all \( r \in [0, 1) \), \( \eta \in T \), and \( j = 1, \ldots, d_\theta \)
- vanishes at \( r = 0 \): For all \( \eta \in T \) and \( j = 1, \ldots, d_\theta \),

\[
\partial_\eta E_P \psi_j(W, \theta_0, \eta) \bigg|_{\eta = \eta_0} [\eta - \eta_0] := D_{0,j}[\eta - \eta_0] = 0.
\]

Heuristically, small deviations in nuisance functions do not invalidate moment conditions.
Results will make use of sample splitting:

- \( \{1, ..., N\} = \) set of all observations;
- \( I = \) main sample = set of observation numbers, of size \( n \), is used to estimate \( \theta_0 \);
- \( I^c = \) auxiliary sample = set of observations, of size \( \pi n = N - n \), is used to estimate \( \eta_0 \);
- \( I \) and \( I^c \) form a random partition of the set \( \{1, ..., N\} \)

Use of sample splitting allows to get rid of ”entropic” requirements and boil down requirements on ML estimators \( \hat{\eta} \) of \( \eta_0 \) to just rates.
Denote
\[ J_0 := \partial_{\theta'} \left\{ E_P[\psi(W, \theta, \eta_0)] \right\} \bigg|_{\theta = \theta_0} \]

Let \( \omega, c_0, \) and \( C_0 \) be strictly positive (and finite) constants, \( n_0 \geq 3 \) be a positive integer, and \( (B_{1n})_{n \geq 1} \) and \( (B_{2n})_{n \geq 1} \) be sequences of positive constants, possibly growing to infinity, with \( B_{1n} \geq 1 \) for all \( n \geq 1 \).

Assume for all \( n \geq n_0 \) and \( P \in \mathcal{P}_n \)

- (Parameter not on boundary) \( \theta_0 \) satisfies (3), and \( \Theta \) contains a ball of radius \( C_0 n^{-1/2} \log n \) centered at \( \theta_0 \)
- (Differentiability) The map \( (\theta, \eta) \mapsto E_P[\psi(W, \theta, \eta)] \) is twice continuously Gateaux-differentiable on \( \Theta \times \mathcal{T} \)
  - Does not require \( \psi \) to be differentiable
- (Neyman Orthogonality) \( \psi \) obeys the orthogonality condition for the set \( \mathcal{T} \subset \mathcal{T} \)
• **(Identifiability)** For all $\theta \in \Theta$, we have
  \[ \| E_P[\psi(W, \theta, \eta_0)] \| \geq 2^{-1} \| J_0(\theta - \theta_0) \| \wedge c_0 \]
  where the singular values of $J_0$ are between $c_0$ and $C_0$

• **(Mild Smoothness)** For all $r \in [0, 1)$, $\theta \in \Theta$, and $\eta \in \mathcal{T}$
  \begin{itemize}
  \item $E_P[\| \psi(W, \theta, \eta) - \psi(W, \theta_0, \eta_0) \|^2] \leq C_0(\| \theta - \theta_0 \| \vee \| \eta - \eta_0 \|_e)^\omega$
  \item $\| \partial_r E_P[\psi(W, \theta, \eta_0 + r(\eta - \eta_0))] \| \leq B_1n \| \eta - \eta_0 \|_e$
  \item $\| \partial_r^2 E_P[\psi(W, \theta_0 + r(\theta - \theta_0), \eta_0 + r(\eta - \eta_0))] \| \leq B_2n(\| \theta - \theta_0 \|^2 \vee \| \eta - \eta_0 \|^2_\epsilon)$
  \end{itemize}
Second key condition is that nuisance functions are estimated “well-enough”:

Let \((\Delta_n)_{n \geq 1}\) and \((\tau_{\pi n})_{n \geq 1}\) be some sequences of positive constants converging to zero, and let \(a > 1\), \(v > 0\), \(K > 0\), and \(q > 2\) be constants.

Assume for all \(n \geq n_0\) and \(P \in \mathcal{P}_n\)

- **(Estimator and Truth)** (i) w.p. at least \(1 - \Delta_n\), \(\hat{\eta}_0 \in \mathcal{T}\) and (ii) \(\eta_0 \in \mathcal{T}\).
  - Recall that “parameter space” for \(\eta\) is \(\mathcal{T}\)
- **(Convergence Rate)** For all \(\eta \in \mathcal{T}\), \(\|\eta - \eta_0\|_e \leq \tau_{\pi n}\)
Theory: Conditions on Estimators of Nuisance Functions (Continued)

- **(Pointwise Entropy)** For each $\eta \in \mathcal{T}$, the function class $\mathcal{F}_{1,\eta} = \{ \psi_j(\cdot, \theta, \eta) : j = 1, \ldots, d_\theta, \theta \in \Theta \}$ is suitably measurable and its uniform entropy numbers obey

$$\sup_Q \log N(\epsilon \| F_{1,\eta} \|_{Q,2}, \mathcal{F}_{1,\eta}, \| \cdot \|_{Q,2}) \leq v \log (a/\epsilon), \quad \text{for all } 0 < \epsilon \leq 1$$

where $F_{1,\eta}$ is a measurable envelope for $\mathcal{F}_{1,\eta}$ that satisfies $\| F_{1,\eta} \|_{P,q} \leq K$

- **(Moments)** For all $\eta \in \mathcal{T}$ and $f \in \mathcal{F}_{1,\eta}$, $c_0 \leq \| f \|_{P,2} \leq C_0$

- **(Rates)** $\tau_{\pi n}$ satisfies (a) $n^{-1/2} \leq C_0 \tau_{\pi n}$, (b) $(B_{1n} \tau_{\pi n})^{\omega/2} + n^{-1/2 + 1/q} \leq C_0 \delta_n$, and (c) $n^{1/2} B_{1n}^2 B_{2n} \tau_{\pi n}^2 \leq C_0 \delta_n$.

Rate of convergence is $\tau_{\pi n}$ - needs to be faster than $n^{-1/4}$

- Same as rate condition widely used in semiparametrics employing classical nonparametric estimators
Let "Double ML" or "Orthogonalized ML" estimator

\[ \hat{\theta}_0 = \hat{\theta}_0(I, I^c) \]

be such that

\[ \left\| E_{n,I}[\psi(W, \hat{\theta}_0, \hat{\eta}_0)] \right\| \leq \inf_{\theta \in \Theta} \left\| E_{n,I}[\psi(W, \theta, \hat{\eta}_0)] \right\| + \epsilon_n, \quad \epsilon_n = o(\delta_n n^{-1/2}) \]

**Theorem (Main Result)**

*Under assumptions stated above, \( \hat{\theta}_0 \) obeys*

\[ \sqrt{n} \Sigma_0^{-1/2} (\hat{\theta}_0 - \theta_0) = \frac{1}{\sqrt{n}} \sum_{i \in I} \bar{\psi}(W_i) + O_P(\delta_n) \rightsquigarrow N(0, I), \]

*uniformly over \( P \in P_n \), where \( \bar{\psi}(\cdot) := -\Sigma_0^{-1/2} J_0^{-1} \psi(\cdot, \theta_0, \eta_0) \) and \( \Sigma_0 := J_0^{-1} E_P[\psi^2(W, \theta_0, \eta_0)](J_0^{-1})' \).
Theory: Attaining full efficiency by Cross-Fitting

- full efficiency not obtained, but can follow Belloni et al (2010, 2012) to do the following:

**Corollary**

*Can do a random 2-way split with \( \pi = 1 \), obtain estimates \( \hat{\theta}_0(I, I^c) \) and \( \hat{\theta}_0(I^c, I) \) and average them*

\[
\hat{\theta}_0 = \frac{1}{2} \hat{\theta}_0(I, I^c) + \frac{1}{2} \hat{\theta}_0(I^c, I)
\]

to gain full efficiency.

**Corollary**

*Can do also a random K-way split \((I_1, ..., I_K)\) of \(\{1, ..., N\}\), so that \(\pi = (K - 1)\), obtain estimates \(\hat{\theta}_0(I_k, I_k^c)\), for \(k = 1, ..., K\), and average them*

\[
\hat{\theta} = \frac{1}{K} \sum_{k=1}^{K} \hat{\theta}_0(I_k, I_k^c)
\]

to gain full efficiency.
Given the split \((I, I^c)\), it is tempting to use \(I^c\) to build a collection of ML estimators

\[
\hat{\eta}_m(I^c), \quad m = 1, \ldots, M
\]

for the nuisance parameters \(\eta\), and then pick the winner \(\hat{\eta}_m(I) (I^c)\) based upon \(I\). This does break the sample-splitting.

- The results still go through under the condition that the winning method has the rate \(\tau_{\pi n}\) such that

\[
\tau_{\pi n} \sqrt{\log M} \to 0.
\]

- The entropy is back, but in a gentle, \(\sqrt{\log M}\) way.
How to Build Orthogonal Scores

Can generally construct moment/score functions with desired orthogonality property building upon classic ideas of Neyman (1958, 1979).

Illustrate in parametric likelihood case.

Suppose log-likelihood function is given by $\ell(W, \theta, \beta)$

- $\theta$ $d$-dimensional parameter of interest
- $\beta$ $p_0$-dimensional nuisance parameter

Under regularity, true parameter values satisfy

$$E[\partial_\theta \ell(W, \theta_0, \beta_0)] = 0, \quad E[\partial_\beta \ell(W, \theta_0, \beta_0)] = 0$$

$$\varphi(W, \theta, \beta) = \partial_\theta \ell(W, \theta, \beta)$$ in general does not possess the orthogonality property
How to Builds Orthogonal Scores in Parametric Likelihood Model

Can construct new estimating equation with desired orthogonality property:

\[ \psi(W, \theta, \eta) = \partial_{\theta} \ell(W, \theta, \beta) - \mu \partial_{\beta} \ell(W, \theta, \beta), \]

- Nuisance parameter: \( \eta = (\beta', \text{vec} (\mu)')' \in T \times D \subset \mathbb{R}^p, \quad p = p_0 + dp_0 \)
- \( \mu \) is the \( d \times p_0 \) orthogonalization parameter matrix
  - True value \((\mu_0)\) solves \( J_{\theta\beta} - \mu J_{\beta\beta} = 0 \) (i.e., \( \mu_0 = J_{\theta\beta} J_{\beta\beta}^{-1} \)) for

\[
J = \begin{pmatrix}
J_{\theta\theta} & J_{\theta\beta} \\
J_{\beta\theta} & J_{\beta\beta}
\end{pmatrix} = \partial_{(\theta', \beta')} \mathbb{E} \left[ \partial_{(\theta', \beta')} \ell(W, \theta, \beta) \right] \bigg|_{\theta=\theta_0; \beta=\beta_0}
\]

- Will have \( \mathbb{E}[\psi(W, \theta_0, \eta_0)] = 0 \) for \( \eta_0 = (\beta_0', \text{vec} (\mu_0)')' \) (provided \( \mu_0 \) is well-defined)
- Importantly, \( \psi \) obeys the orthogonality condition: \( \partial_{\eta} \mathbb{E}[\psi(W, \theta_0, \eta)] \bigg|_{\eta=\eta_0} = 0 \)
- \( \psi \) is the efficient score for inference about \( \theta_0 \)
How to Builds Orthogonal Scores in Moment Conditions Models

More generally, can construct orthogonal estimating equations as in the semiparametric estimation literature.

For example, can proceed by projecting score/moment function onto orthocomplement of tangent space induced by nuisance function:


Orthogonal scores/moment functions will often have nuisance parameter \( \eta \) that is of higher dimension than “original” nuisance function \( \beta \).

- Also see in partially linear model where nuisance parameter in orthogonal moment conditions involve two conditional expectations.
Example 1. ATE in Partially Linear Model

Recall

\[ Y = D\theta_0 + g_0(Z) + \zeta, \quad \mathbb{E}[\zeta \mid Z, D] = 0, \]
\[ D = m_0(Z) + V, \quad \mathbb{E}[V \mid Z] = 0. \]

Base estimation on orthogonal moment condition

\[ \psi(W, \theta, \eta) = ((Y - \ell(Z) - \theta(D - m(Z)))(D - m(Z)), \quad \eta = (\ell, m). \]

Easy to see that

- \( \theta_0 \) is a solution to \( \mathbb{E}_P \psi(W, \theta_0, \eta_0) = 0 \)
- \( \partial_\eta \mathbb{E}_P \psi(W, \theta_0, \eta) \bigg|_{\eta = \eta_0} = 0 \)
Consider a treatment $D \in \{0, 1\}$. We consider vectors $(Y, D, Z)$ such that

$$Y = g_0(D, Z) + \zeta, \quad \mathbb{E}[\zeta \mid Z, D] = 0, \quad (4)$$
$$D = m_0(Z) + \nu, \quad \mathbb{E}[\nu \mid Z] = 0. \quad (5)$$

The average treatment effect (ATE) is

$$\theta_0 = \mathbb{E}[g_0(1, Z) - g_0(0, Z)].$$

The average treatment effect for the treated (ATT)

$$\theta_0 = \mathbb{E}[g_0(1, Z) - g_0(0, Z) \mid D = 1].$$

- The confounding factors $Z$ affect the $D$ via the propensity score $m(Z)$ and $Y$ via the function $g_0(D, Z)$.
- Both of these functions are unknown and potentially complicated, and we can employ Machine Learning methods to learn them.
Example 2 Contuned. ATE and ATT in the Heterogeneous Treatment Effect Model

For estimation of the ATE, we employ

$$\psi(W, \theta, \eta) := \theta - \frac{D(Y - \eta_2(Z))}{\eta_3(Z)} - \frac{(1 - D)(Y - \eta_1(Z))}{1 - \eta_3(Z)} - (\eta_1(Z) - \eta_2(Z)), \quad (6)$$

$$\eta_0(Z) := (g_0(0, Z), g_0(1, Z), m_0(Z))',$$

where $\eta(Z) := (\eta_j(Z))_{j=1}^3$ is the nuisance parameter. The true value of this parameter is given above by $\eta_0(Z)$.

For estimation of ATT, we use the score

$$\psi(W, \theta, \eta) = \frac{D(Y - \eta_2(Z))}{\eta_4} - \frac{\eta_3(Z)(1 - D)(Y - \eta_1(Z))}{(1 - \eta_3(Z))\eta_4} + \frac{D(\eta_2(Z) - \eta_1(Z))}{\eta_4} - \theta \frac{D}{\eta_4},$$

$$\eta_0(Z) = (g_0(0, Z), g_0(1, Z), m_0(Z), E[D])',$$

(7)
Example 2 Continued. ATE and ATT in the Heterogeneous Treatment Effect Model

It can be easily seen that true parameter values $\theta_0$ for ATT and ATE obey

$$E_P \psi(W, \theta_0, \eta_0) = 0,$$

for the respective scores and that the scores have the required orthogonality property:

$$\partial_\eta E_P \psi(W, \theta_0, \eta) \bigg|_{\eta = \eta_0} = 0.$$

We use ML methods to obtain:

$$\hat{\eta}_0(Z) := (\hat{g}_0(0, Z), \hat{g}_0(1, Z), \hat{m}_0(Z))',$n

$$\hat{\eta}_0(Z) = (\hat{g}_0(0, Z), \hat{g}_0(1, Z), \hat{m}_0(Z), \mathbb{E}_n[D]).$$

The resulting “double ML” estimator $\tilde{\theta}_0$ solves the empirical analog:

$$\mathbb{E}_n, \iota \psi(W, \tilde{\theta}_0, \hat{\eta}_0) = 0, \tag{8}$$

and the solution $\tilde{\theta}_0$ can be given explicitly since the scores are affine with respect to $\theta$. 
Example 3. LATE and LATTE in Heterogeneous Treatment Effect Models

- LATE can be written as a ratio of ATE of a binary instrument on $D$ and $Y$, so can use Example 2 to estimate each piece.
- Similar construction works for LATTE.
- By defining $Y^* = 1(Y \leq t)$ can study Distributional and Quantile Treatment Effects.
- See ”Program Evaluation ...” paper for details.
Example 4. Moment Condition Models

Very common framework in structural econometrics.

- See Chernozhukov, Hansen, Spindler ARE, 2015 for parametric case
- See “Program Evaluation ...” (Econometrica, 2016) for semi-parametric case.
- See the paper with Whitney on locally robust moments.
Empirical Example: 401(k) Pension Plan

Follow Poterba et al (97), Abadie (03). Data from 1991 SIPP, \( n = 9,915 \)

- \( Y \) is net total financial assets
- \( D \) is indicator for working at a firm that offers a 401(k) pension plan
- \( Z \) includes age, income, family size, education, and indicators for married, two-earner, defined benefit pension, IRA participation, and home ownership

\( D \) is plausibly exogenous at the time when 401(k) was introduced

Controlling for \( Z \) is important due to 401(k) mostly offered by firms employing mostly workers from middle and above middle class (Poterba, Venti, and Wise 94, 95, 96, 01)
## Empirical Example: 401(k)

### Table: Estimated ATE of 401(k) Eligibility on Net Financial Assets

<table>
<thead>
<tr>
<th>Method</th>
<th>RForest</th>
<th>PLasso</th>
<th>B-Trees</th>
<th>Nnet</th>
<th>BestML</th>
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</thead>
<tbody>
<tr>
<td><strong>A. Part. Linear Model</strong></td>
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<tr>
<td>ATE</td>
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<td>8984</td>
<td>8612</td>
<td>9319</td>
<td>8922</td>
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<td><strong>B. Interactive Model</strong></td>
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</table>

Estimated ATE and heteroscedasticity robust standard errors (in parentheses) from a linear model (Panel B) and heterogeneous effect model (Panel A) based on orthogonal estimating equations. Column labels denote the method used to estimate nuisance functions. Further details about the methods are provided in the main text.
We provide a general set of results that allow $\sqrt{n}$-consistent estimation and provably valid (asymptotic) inference for causal parameters, using a wide class of flexible (ML, nonparametric) methods to fit the nuisance parameters.

**Three key elements:**

1. Neyman-Orthogonal estimating equations
2. Fast enough convergence of estimators of nuisance quantities
3. Sample splitting
   - Really eliminates requirements on the entropic complexity on the realizations of $\hat{\eta}$
   - Allows establishment of results using only rate conditions, not exploiting specific structure of ML estimators (as in, e.g., results for inference following lasso-type estimation in full-sample)
Thank you!

References.

  with Alexandre Belloni, I. Fernandez-Val, Christian Hansen

• "Double Machine Learning for Causal and Treatment Effects"
  ArXiv 2016, with Denis Chetverikov, Esther Duflo, Christian Hansen, Mert Demirer, Whitney Newey