

# Bayesian thoughts

As a school student, **Dennis Lindley** had wanted to be an architect, but advice from his teachers led him to work on statistics instead. Later in this issue he focuses on the prior probability, but here, in communication with Helen Joyce, he offers a more personal description of his life as a Bayesian.

## How did you start your career in statistics?

My ambition at school was to be an architect and, more specifically, to go to the Bartlett School of Architecture at University College London (UCL). After my Higher School Certificate (now A-levels) my teachers told me I would be a better mathematician than architect and persuaded me to apply to Cambridge. I got an exhibition, later senior scholarship, at Trinity College, and was there from 1941 to 1943, doing part ii mathematics.

I became a statistician to avoid military service. In late 1942 some of us were told that if we took an additional course in statistics we could have a post in the Ministry of Supply, working on statistical problems in production. Work there was most enjoyable and, apart from a return to Cambridge for part iii in 1946–1947, I had a permanent civil service post, moving to the National Physical Laboratory in 1945.

## And after that?

Then, much to my surprise, in 1948 I was invited to apply for a demonstrator's post at Cambridge and was there from 1948 to 1960, eventually becoming Director of the Statistical Laboratory, not through any merit, but because people senior to me left.

In 1960 I went to Aberystwyth, setting up a new department. This was in the days when universities were relatively rich. In 1967 I went as Head of Department to UCL, which was intellectually stimulating but socially less pleasant. Although some people conveniently left, allowing Bayesian appointments to be made, it was never possible to capture the Bayesian ethos of Aberystwyth. I also made an administrative error and allowed myself to be persuaded to include Computer Science in the department. In those days, UCL appointed a Head of Department and largely left him to get on with the job, which is fine, but I do not like administration.

In 1977, trying to persuade two people to retire, I read the new pension arrangements carefully and found that I could retire, so I did. From then until 1987 I travelled the world as an "itinerant" scholar and had a wonderful time.

## How did your time at these various institutions compare?

Cambridge was splendid because the students were of such a high quality. Aberystwyth was enjoyable because one really belonged to a community. I used to joke that I knew what went on in the university as much through my wife's socialising as I did officially. UCL was different as the people sped to the suburbs in the evenings and vacations, the community atmosphere being much reduced. Despite being born in London, I do not like cities. Nevertheless, the intellectual life in UCL, and London generally, was excellent.

## You are well known as a proponent of Bayesian methods. Was there a pivotal moment when you became a Bayesian?

No. The adoption of Bayesian ideas was gradual and modifications continually arose. My first introduction was through attendance at the lectures of Harold Jeffreys (1947). Then came my own proof of the foundations result<sup>1</sup>, followed by a year with Jimmie Savage (1954–1955) who had a far better proof.

I had developed the Bayesian view as an attempt to provide a firm structure for what I saw as modern statistics, due to Fisher and to Neyman and Pearson. Around 1960 I came to the realisation that it did not bolster these methods but rather contradicted them. Significance tests and confidence intervals were

**"We need to restore reasoning to its proper place in modern thought"**



unsound. Later in the 1970s it was shown that ideas using improper priors were unsound and only proper, subjective priors would work. Learning has been a continual process.

## Is the work on foundations worthwhile?

Most emphatically “yes”. Work on the foundations in the 20th century provided, first, an extension of the work of Bayes and Laplace, leading to the concept of maximisation of expected utility (MEU), but secondly, and more importantly, showed that any method that was not Bayesian was unsatisfactory, in that some Bayesian method was necessarily better. In other words, we could restrict attention to Bayes.

Furthermore, MEU was an operational method that could be used and made statistics function like Newtonian mechanics. In that field, any problem can be solved by writing down Newton’s equations and solving them. Now, in inference, one had a similar routine using the calculus of probability. With these foundations in place, statistics ceased to be a series of “ad hoceries” and became, for the first time, an organised subject.

The development of the foundations showed that any measure of uncertainty had to be additive, as is probability. Likelihood, on the other hand, is not additive and so likelihood, on its own, cannot be a satisfactory measure of uncertainty.

Nowadays the foundations do not need to be studied, at least in such detail, because the central problem that the 20th century tackled has been solved.

## Would computers have made a difference?

Yes. I have wasted a lot of research effort trying to solve, by analytical methods, situations that nowadays yield to Markov chain Monte Carlo methods: for example, analysis of data with unequal variances or with  $t$ -distributions. Many of Fisher’s ingenious and valuable ideas were essentially solutions to computational problems caused by the limitations of the Brunsviga. Bayes only became really operational with the arrival of the modern computer.

## Are there dangers in Bayesian methods?

The main danger is that they will be used automatically. In the application of Bayesian methods, you first need to assign some basic probabilities (and perhaps utilities) to give the problem structure; then the computer can derive other probabilities. One must *think* about the basic values and it is not usually satisfactory to use a normal density and non-informative priors. You must think about the real quantities involved, like temperature or blood pressure, and not about symbols that represent them. This distinction between the thinking you and the unthinking, calculating personal computer is essential.

**“Ultimately, in my extreme view, all reasoning reduces to probability calculations”**

## Obviously, there are many statisticians who do not support Bayesian ideas. Does this irritate you?

I am easily irritated. What really annoys me is the lack of knowledge of MEU amongst Fellows of our Society. As a young man, I had thought that anyone reading a mathematical proof, and finding no error therein, had only two choices: accept and use the result, or find a flaw in the premises. They don’t do this. Savage, around 1960, is reported to have said to his colleagues: “In 1954 I proved that the only sound methods were Bayesian; yet you continue to use non-Bayesian ideas without pointing out a flaw in either my premises or my proof, why?”. And answer came there none. Another colleague described his experience as like getting on the Bayesian bus but, when he found where he was going, he got off.

## Any regrets about your career?

I wish I had not been so critical of those who opposed Bayesian ideas!

## Do you feel that there are areas of statistical research which are unnecessary or misguided?

I feel most of the work on multiple comparisons is misguided. If an experiment is performed with a factor at many levels, for example, an agricultural trial with several varieties of wheat, then a standard  $t$ -test of the variety with the largest sample mean against that with the smallest will clearly be invalid because of the varietal selection. A whole book has been written on the problem, yet its solution is immediate. The true mean yields form a parameter set with a posterior distribution, given the data. For any two varieties, their bivariate, marginal distribution can be calculated: problem solved.

## In which areas would you say there is a need for more research?

In the assessment of probabilities. What is your probability that Blair will lead the Labour Party into the next election?

I have emphasised the need to *think* when using Bayes. Yet we devote too little effort to doing so, and we know almost nothing about assessing probabilities for four, or more, quantities, though progress has been made using various tree structures. This is despite the development of powerful methods for handling large data sets with many variables. Here the computer has overtaken the thinker, whereas in my younger days the thinker had to deal with a poor computer.

## And where, in everyday life, would you like to see more use of statistics?

One does not encounter the law every day, but I think there is a lot of scope for the use of statistical ideas in law. The field has similarities to statistics in that it attempts to reduce uncertainty in a court of law by the provision of evidence. Consequently statistical ideas should be used in court and have already been used in the analysis of forensic data. But there are other areas to explore. Thus I do not think a jury should be required to decide guilty or innocent; they should provide their probability of guilt. The judge can then apply MEU by incorporating society’s

utility. Hutton could usefully have used some probability. A lawyer and I wrote a paper on the evidential worth of failure to produce evidence.

### How can we make statistics more popular, or more accessible?

I think this is part of a wider problem. Statistics is reasoning using numbers. What we need to do is to restore reasoning to its proper place in modern thought; to make reasoning more popular, to persuade people to be rational and to eschew irrational ideas and prejudices. We need to get rid of the mumbo-jumbo that pervades the media. I am implacably opposed to faith elements in all religions but especially to the fundamentalists who have abandoned reason completely.

I object to the excessive use of emotion: thus the US election later this year will largely be based on whether Bush has better presence than his opponent; and Blair is popular because he has a charming grin, even though he has betrayed the Labour Party.

Emotion has its place, expressed in the form of utility, and incorporated into MEU. Ultimately, in my extreme view, all reasoning reduces to probability calculations.

### And what of the image of the statistician?

Life is a balance between reason and emotion. MEU reflects this in the combination of probability, as the reasoning tool, and utility as the numerical expression of emotion. Statisticians, at least in their professional capacity, concentrate on reasoning and are therefore often thought of, by the more artistic, emotional members of society, as being dry. That is why, at a party, it is best to keep off statistics. A female colleague tells how she used to dread being asked at a party what she did for fear the questioner would be frightened off.

### So, if you were at a party, how would you convince people that statistics was more than just "boring numbers"?

Simpson's paradox. It is easy to produce data, for example a medical trial, which conclusively establishes that treatment A is better than treatment B. Now take that part of the data that deals with the men who took part in the trial. It can happen that B is clearly better than A, reversing the judgment. Similarly, for the women, B wins. So here is a treatment B,

that is good for the men, good for the women, and bad for all of us. When a journalist writes that A is better than B, single jabs are better than measles, mumps and rubella, remember Simpson.

I have found the paradox to be an excellent way of persuading people that statistics is not straightforward.

### In your view, what is the most important statistical method developed during your career?

The likelihood principle. I have criticised likelihood above but, in conjunction with the prior, it plays a dominant role in inference.

Savage tells a good story about the principle. When he was first told it by George Barnard, he expressed surprise that anyone as brilliant as George could say something so ridiculous. Later he came to wonder how anyone could deny something so obviously correct.

### And what would you say is the most important book on statistics?

de Finetti's two-volume work. I have seriously suggested that all statisticians should give up research for two years and read one volume a year. It is a difficult book but properly understood it could change your life.

de Finetti is my hero. Others would say Fisher is theirs. What is important is to remember that heroes make mistakes; even Homer nods. Recently someone said that fiducial probability was sound because Fisher proposed it; this despite the fact that I proved it unsound. (I disagreed with de Finetti over utility.)

**"Probability is the only satisfactory way to reason in an uncertain world"**

### Does this mean that the foundations are only important to academics? What about the many statisticians who are not working in research—can they do just fine without?

Everyone, not just statisticians, should know about probability because probability is the *only* satisfactory way to reason in an uncertain world. Everyone concerned should have thought about the probability that, in September 2002, Iraq had weapons of mass destruction. Probability is at the heart of the Bayesian

method, which describes how your uncertainty changes with data. And any statistical technique that is not Bayesian can be improved by a Bayesian method.

We all need a framework within which to think and Bayesian ideas provide one, highlighting what are the important aspects that need to be considered and explaining how they can be combined to provide an answer. For example, a risky situation contains two elements: the uncertainties of the outcomes, and the desirabilities of the outcomes. The former are described by probabilities, the latter by utilities; the combination is effected through expected utility, the optimum course of action being to make it as large as possible.

Over Iraq, had the US stated its utilities, we would have seen whether they were interested in democracy, oil or profit for US firms.

### What is the greatest challenge facing statistics today?

There are many but one that interests me is persuading the public, and even many members of our profession, of the value of utility. We have tended to use a few limited loss functions without carefully thinking what the losses, or utilities, really are. We have not always appreciated that we can make a contribution to decision-making and can not only present the data informatively but can explain how those data can be used to assist action.

### If you had to choose one piece of work you were most proud of, which would it be?

My book, *Making Decisions* (Wiley, 1985) has perhaps the greatest social value of anything I have written. However, I usually turn this question into a joke and cite my contribution to *The Oxford Companion to Wine*.

#### Reference

1. Lindley, D. V. (1953) Statistical inference (with discussion). *Journal of the Royal Statistical Society, Series B*, 15, 30–76.

Dennis V. Lindley is Emeritus Professor of Statistics at University College London and is widely recognised as one of the most influential statisticians of his generation. The Lindley prize, awarded in recognition of innovative research in Bayesian statistics and administered by the International Society for Bayesian Analysis, recognises the "impact and importance of [Lindley's] work in the foundations, theory and application of Bayesian statistics". He was awarded the Guy Medal in Silver by the Royal Statistical Society in 1968 and the Guy Medal in Gold in 2002.