Democratic Policy Decisions with Decentralized Promises Contingent on Vote Outcome*

Ali Lazrak† and Jianfeng Zhang‡

November 6, 2024

Abstract

We study pre-vote interactions in a committee that enacts a welfare-improving reform through voting in the presence of heterogeneous utilities. Committee members use decentralized irrevocable promises of non-negative transfers contingent on the vote outcome to influence the voting behaviour of other committee members. Equilibrium transfers require that no coalition can deviate in a self-enforcing manner that benefits all its members (*Strong Nash*) and minimize total transfers. We show that equilibria exist, are indeterminate, efficient, and involve transfers from high- to low-utility members. Equilibrium transfers prevent reform opponents from persuading less enthusiastic reform supporters to vote against the reform. Transfer recipients can be reform supporters.

Keywords: Transfer promises contingent on vote outcome, political failure, majority coercion, direct democracy, strong Nash, total transfer minimization

JEL Codes: D70

*We thank the students from the UBC reading group on political economy and finance in the summer 2022, and the participants in the UBC VSE Micro lunch workshop, the USC colloquium, the Bachelier Congress, the Stockholm School of Economics conference in memory of Tomas Björk, ESEM 2023 (Barcelona), the BIRS workshop in Banff, the Chicago Conference on Stochastic Analysis and Financial Mathematics, The Game Theory Days (UM6P, Rabat), The CUHK colloquium. We thank William Cassidy, Alvin Chen, Lorenzo Garlappi, Navin Kartik, Chad Kendall, Henri Klintebaeck, Antonin Macé, Nicola Persico, Dean Spencer, Tristan Tomala for comments. Ali Lazrak gratefully acknowledges the support from The Social Sciences and Humanities Research Council of Canada. Jianfeng Zhang gratefully acknowledges the support from NSF grants DMS-1908665 and DMS-2205972.

†University of British Columbia, Sauder School of Business, email: ali.lazrak@sauder.ubc.ca

‡University of Southern California, mathematics department, email: jianfenz@usc.edu

1 Introduction

Voting in political elections is often regarded as a moral duty, and trading votes for money or favors can be seen as abhorrent. Similarly, voting rights in corporations are a crucial way for shareholders to express their opinions on decisions that impact their economic ownership. Decoupling voting rights from economic ownership may, therefore, undermine the ideal of allocative efficiency in capital markets. Despite these negative connotations, vote trading remains a widespread practice that can take various accepted forms within legal and societal norms. For example, legislative logrolling is a common practice where politicians exchange their votes on particular issues in return for votes on other issues.[1] In the case of corporate votes, activist investors may borrow shares for a nominal fee and use them to vote in favor of their own private agendas.[2] Although commonly used in practice, the normative properties of vote trading are not well understood. In a recent survey, Casella and Macé [2021] highlighted that "Given the prominence of vote trading in all groups' decision-making, it is very surprising how little we know and understand about it." [3]

[1]Early evidence of logrolling motivated by greed was found in the British parliament during the 1840s "railways mania." Railway companies had to petition Parliament for a Private Act allowing them to begin construction of their lines. During the railway mania, an early case of a technology bubble, substantial funds were drawn from optimistic investors, including Members of Parliament (MPs). Parliamentary rules prevent MPs from directly voting on private acts concerning companies at arm's length, aiming to safeguard against personal interests influencing the approval of projects. Even so, evidence suggests that vote trading occurred between MPs, prioritizing individual interests (see Esteves and Geisler Mesevage [2021]). More recent evidence of logrolling based on personal connection has been reported in the US Senate (Cohen and Malloy [2014]) and in the US Congress (Battaglini et al. [2023]).

[2]Hu and Black [2005] offer an overview of the "decoupling techniques" that are used in practice to unbundle the common shares' economic interest from voting rights.

[3]Vote trading has complex welfare implications, with both known and unknown effects. It can improve efficiency by allowing voters to express their preferences more precisely than with a simple binary vote (Buchanan and Tullock [1962] and Coleman [1966]). It can also hinder efficiency by imposing negative externalities on others (Downs [1957]).

In this paper, we evaluate one form of vote trading, where prior to casting their votes, the members of a committee make decentralized transfer promises contingent on the vote outcome. The committee, whether it is a legislative body or the shareholder base of a corporation, decides whether to enact a reform or reject it and retain the status quo. In our analysis, the committee decision is based on a quota rule, such as simple majority, supermajority, or unanimity. Committee members are fully informed about all relevant information, yet they disagree on their preferred alternatives because they derive different utilities from the passage of the reform. We assume that the reform is socially optimal in that the sum of utilities for the reform net of that for the status quo is positive. Before voting, however, members can freely make credible and enforceable transfer promises contingent on the committee's decision. The promises are unconstrained and involve coalitions of any size ranging from a pair to the entire committee. The promises alter the incentives to vote, but voters retain control of their voting rights and cast their votes to maximize self-interest.

The transfer promises from our model capture some elements of the corporate bankruptcy proceedings where creditors and sometimes shareholders are asked to vote on a proposed plan of reorganization or liquidation, as opposed to resolving the issues through court intervention. The wedge between the expected court rulings' terms and the proposal's terms represents promises of transfers between stakeholders, as we envision in our model. Our model also reflects a common practice in legislative bodies, such as the US Congress, where bills are frequently amended before a final vote. The final bill bundles the initial bill and the amendments, which we view as transfers between congressmen that are only received when the bill passes.[4] More generally, the promises can represent favors (e.g., logrolling), legislative amendments,

[4]For example, the Patient Protection and Affordable Care Act (commonly known as Obamacare) was passed by the US Congress in 2010. During the legislative process, the bill was amended in order to gain enough support from both Democrats and Republicans. Several concessions to moderate Democrats and Republicans were made, including removing a public option and scaling back the scope of the bill.

terms of liquidation, monetary payments, or any commitment to certain future actions that increase the advantage to the recipients of the promises.

To undertake our evaluation of the practice of promises contingent on vote outcome, we formulate a two-stage game that we now describe informally. In the second stage, committee members take the transfers as given and vote for or against the reform. Since the vote is binary and the heterogeneity is single dimensional, it is natural to assume sincere voting in the second stage game (Black [1958]). In the first stage, players *simultaneously* announce transfer promises functions based on the vote outcome. The transfer promises are non-negative, enforceable and credible. Once the promises are made but before the voting occurs, we allow players to deviate in a coordinated manner. Committee members are given the opportunity to form *blocking coalitions*, enabling them to influence other committee members and coordinate their actions to achieve a more favorable outcome for all involved. However, they are restricted from retracting existing promises; they may only increase the transfer promises already in place or establish new ones. This assumption is appropriate for committees, like respected political institutions in democracies, where breaking a promise carries considerable reputational costs.[5] We prohibit the formation of blocking coalitions in the first stage game and assume, moreover, that the total transfers are minimized[6]. Our political equilibrium is thus a strong Nash equilibrium of a certain game (Aumann [1959]),

[5]Alesina [1988] and Alesina and Spear [1988] model electoral competition as a dynamic game with repeated interactions and demonstrate that the equilibrium of this game leads politicians to fulfill their promises. Their model illustrates how dynamic interactions can sustain commitment, showing that the behavior of not retracting can emerge as an equilibrium outcome rather than being imposed as an assumption.

[6]By minimizing transfer promises, we reduce the unmodeled transaction costs associated with them. For instance, in politics, limiting amendments to a bill helps avoid deviation from its original purpose, especially if many changes might displease constituents or donors. Similarly, in logrolling or corporate liquidation, promises of future transfers can create uncertain costs and benefits. Minimizing these promises reduces uncertainty and related costs, and in some contexts, such as 19th-century British parliament during the "railways mania," it can also minimize the risk of detection.

which in addition minimizes the total transfers by all committee members. We call such equilibria *Strong Minimal equilibria* or "SM" equilibria.

In the absence of transaction costs and informational frictions, a "Coasean intuition" suggests that committee members should be able to contract to achieve an efficient outcome (Coase [1960], Stigler [1966]). However, this intuition has its limitations within our framework. While it's natural to model stable contracts using the strong Nash equilibrium in environments where coordination among group members is feasible, the criteria for achieving a strong Nash equilibrium is known to be quite demanding. Their existence is not always guaranteed, meaning that we cannot always ensure efficiency will be achieved, and the Coasean intuition may not necessarily hold (see, e.g., Medema [2020]). A key contribution of this paper is to hold the Coasean intuition to scrutiny in our environment. In doing so, we provide an instance where strong Nash equilibria do exist and where we can visualize them and contrast them with Nash equilibria (see Panel A of Figure 1). Moreover, we offer novel predictions about the expected amounts and directions of the equilibrium transfers for the environments where our assumptions on the transfer promises are valid.

Our first result is to provide a complete characterization of the Strong Nash equilibria of our two stages game. The characterization allows a detailed analysis of these equilibria. The first insight is that multiple Strong Nash equilibria exist and achieve the social optimum, meaning the reform is enacted. The efficiency result is a natural outcome, given that the entire committee has the ability to deviate, which implies that any strong equilibrium, if it exists, must be efficient. A more significant result is the identification of a set of relevant assumptions that guarantee the existence of strong equilibria. The key assumptions for ensuring existence are the irrevocability of promises and their dependence on the vote outcome. In the absence of these conditions, we show that the very existence of Strong Nash equilibria may be compromised. Our existence result indicates that the practice of transfer promises contingent on vote outcome promotes efficiency when committee members can coordinate their actions and commit to not retract their promises. We also show that SM equilibria exist and are indeterminate: while reducing the set of strong equilibria, the minimal total transfer assumption does not imply uniqueness. The indeterminacy arises from the multiple ways to divide the total transfers among the promisers and to distribute the promises among the recipients. However, the assumption of minimality implies that all equilibria share common characteristics. Specifically, in all equilibria SM, there is a critical voter who is a reform supporter, such that the promisers have weakly larger utilities than that member while the promises recipients have lower utilities than that member. In all SM equilibria, the transfer promises flow from committee members with higher utilities to those with lower utilities. Promisers are always reform supporters. The recipients of the promises, however, can be either reform opponents or supporters, depending on the distribution of utilities and the quota rule.

When the reform is defeated in the absence of transfer, all SM equilibria share the common characteristic of being in "the reaching across the aisle" type where the promisers support the reform while the recipients of the promises oppose it. In these equilibria, reform supporters compensate the reform opponents with the total disutility that they experience when the reform passes. Thus when committee members are more polarized, we expect the equilibrium total transfer to increase. Recently, *earmarking*, which refers to the practice of legislators allocating funds for specific projects in their district or state, has been revived in the US Congress.[7] Given that earmarks provide lawmakers with incentives to vote for legislation they may not support otherwise, they can be broadly interpreted as transfers within the framework of our model. The observed increase in polarization within the US Congress, coupled with the restoration of earmarking, aligns with the predictions of our model.

In the alternative scenario where reform supporters possess sufficient voting power to enact the reform, blocking coalitions can exist where opponents of the reform entice reform supporters with the weakest utilities, with transfer promises contingent

[7]https://www.science.org/content/article/congress-restores-spending-earmarks-rules-remove-odor

on defeating the reform. These promises can persuade weakly motivated supporters to switch stances on the reform. When these blocking coalitions are present, transfer promises made by reform supporters with the highest utility become crucial. These "higher order" promises help prevent the reform from being defeated after such coalitions form. We also show that when the reform is defeated without transfers, SM equilibrium transfers can be of the "circle the wagon" type. This happens in equilibria where reform supporters with strong utility make some transfers to other reform supporters with lower utility.[8]

Related literature. The primary contribution of this paper is to demonstrate the feasibility of assessing the practice of transfer promises based on voting outcomes. We focus on a decentralized setting where voters can coordinate their actions when making promises of transfers to one another and when retracting these promises once they are established is not an option. Our results intersect with three streams of literature.

First, our findings relate to the literature on political failures, which aims to identify inefficiencies in political operations and achieve Pareto improvements (see, e.g., Becker [1958] and Wittman [1989]). By demonstrating the success of transfer promises contingent on vote outcome, our study contributes to this literature by reporting a formal mechanism to improve the overall efficiency in collective decisions when the problem of majority coercion is an important concern. More broadly, inefficiencies can arise in general games with externalities, such as the Prisoner's Dilemma. Jackson and Wilkie [2005] demonstrated that, in the absence of coordination among players, inefficiencies

[8] In these equilibria, when a committee operates under majority rule, reform supporters who are close to the median voter receive transfers from the strongest reform proponents. An example of this dynamic can be seen in the U.S. Senate, which is currently closely divided between Republicans and Democrats. Senator Joe Manchin, a moderate Democrat from West Virginia, exemplifies this situation. His support is frequently pivotal for advancing legislation. For instance, Manchin played a crucial role in shaping and passing the Inflation Reduction Act. His influence on the bill is described in https://www.politico.com/newsletters/playbook/2022/08/08/how-it-really-happened-the-inflation-reduction-act-00050279.

can arise when they engage in transfers before participating in these games (See also Ellingsen and Paltseva [2016]). They show in a very general context that Nash pre-stage contracting may not always lead to efficient outcomes. Our framework differs from Jackson and Wilkie [2005] in two key ways: 1) players are allowed to retract the promises in place in their model, whereas our promises are irrevocable, and 2) they consider equilibria that are robust to Nash unilateral deviations, whereas we consider equilibria that are robust to strong Nash coalitional deviations. These differences explain why our results on efficiency differ from theirs. Importantly, our approach provides a complete characterization of equilibria, including predictions of the expected transfers. Moreover, our analysis is specifically tailored to voting games and addresses the political failure of majority coercion, an aspect not covered in Jackson and Wilkie [2005].[9]

Second, we contribute to the literature on vote trading. Despite the significant strides made in the 1960s and 1970s, the subject of vote trading has somewhat faded from scholarly focus in recent decades. A pivotal argument in the development of this literature is that the voting externality can make vote trading undesirable (Riker and Brams [1973]). However, mechanisms such as vote storing (Casella [2012]) and quadratic voting (Eguia et al. [2023]) have been shown to enhance efficiency, offering a more favorable perspective on the practice‘. Casella and Palfrey [2019] also provide a positive perspective on vote trading. They consider committee settings where members vote on multiple issues and sequentially trade votes on one issue in exchange for votes on other issues.[10] We extend this literature by examining a decentralized framework, similar to that of Casella and Palfrey [2019], but focusing on trading through *simultaneous*

[9] While we focus on the strong Nash equilibria, we nonetheless characterize the Nash equilibria of our game and found that the outcome may be inefficient as in Jackson and Wilkie [2005].

[10] Philipson and Snyder [1996] also offer a more positive result on vote trading in the context of an organized centralized vote market where the vote buyers could be party leaders or committee chairs. See also Xefteris and Ziros [2017] for a similar message under different assumptions on the market for votes.

promises. We view this addition as important because many pre-vote interactions in corporate votes, referenda, or political elections can be thought of as transfer promises contingent on the vote outcome.

Finally, we contribute to the literature on promises for political decisions, e.g., Myerson [1993], Groseclose and Snyder [1996], Dal Bo [2007], Dekel et al. [2008], Dekel et al. [2009], and more recently Chen and Zápal [2022], Louis-Sidois and Musolff [2024] and, Domènech-Gironell and Xefteris [2024]. This literature has largely concentrated on models with political representatives and revolves around leaders vying for votes by making pledges or campaign promises. Within this framework, much attention has been devoted to leaders' capacity to utilize budgetary resources to manipulate vote results and gain profits while in office. Through varying assumptions about the game structure, these studies have made significant advancements with an emphasis on the sequencing of promises. Our study offers a different perspective by investigating the potential advantages of outcome-contingent promises in a decentralized setting. In our context, promises happen simultaneously within the committee (or the electorate) and without a designated leader. Rather than examining this practice solely within the political agency framework, we demonstrate how pre-vote interactions through promises can facilitate agreement and drive efficient outcomes in direct democracies. By exploring the potential benefits of vote-contingent promises in this distinct context, our research underscores the need for a broader examination of this practice.

The paper proceeds as follows. Section 2 sets forth the general setting, Section 3 defines Nash equilibria and strong Nash equilibria. Section 4 characterizes and contrasts both type of equilibria, Section 5 defines the Strong minimal (SM) equilibria and shows their existence, indeterminacy and implications. We illustrate more detailed implications of SM equilibria in Section 6 and discuss the assumptions and extensions of the model in Section 7. We conclude in Section 8. The appendix provides the proofs of the paper's propositions. The supplemental appendix presents additional generalizations related to the discussions in Sections 6 and 7.

2 The model

We consider a committee $\mathbb{I} = \{1, \cdots, I\}$ of I members or players. The game has two stages. In the first stage, players make decentralized promises of transfers. In the second stage, players take the transfers as given and engage in voting. We now describe in detail the game starting with the second stage.

Stage 2: The voting game. In the second stage game, a collective decision on whether to accept a reform or reject it in favor of the status quo is taken by the committee. Each player i must vote for the reform ($v_i = 1$) or against it ($v_i = 0$). We consider the pure strategies space $X = \{0, 1\}$ and the associated set of strategy profiles X^I. We denote by v_i and $v = (v_1, .., v_I)$ generic elements of X and X^I. The committee is ruled by a quota rule with threshold $\kappa \in \mathbb{I}$ whereby the reform is enacted if at least κ committee members vote in favor. The threshold for adopting the status quo is denoted as $\widehat{\kappa}$ where $\widehat{\kappa} := I - \kappa + 1$. The reform is defeated if the number of reform opponents is greater than or equal to $\widehat{\kappa}$. We denote the set of vote outcomes by $\mathcal{O} = \{R, S\}$, where $O = R$ (resp. $O = S$) indicates that the reform is adopted (resp. defeated). When the strategy profile v is played, the vote outcome is given by $O = R$, when $\sum_i v_i \geq \kappa$ and, $O = S$, otherwise.

In the absence of transfers, player i's utility is a function $U_i : X^I \to \mathbb{R}$ defined by

$$U_i(v) := \begin{cases} u_i & \text{if} \quad \sum_i v_i \geq \kappa; \\ 0, & \text{otherwise;} \end{cases} \tag{1}$$

where we have normalized the (cardinal) utility derived from the status quo for each voter to 0 and denote u_i the utility experienced by voter i when the reform is adopted. We order the utilities $u_1 \leq \cdots \leq u_I$ and denote the utility vector by $u = (u_1, \cdots, u_I)$. Voters are divided in their stance on the reform and we denote by n the number of reform opponents with $1 \leq n < I$. We denote the coalition of reform opponents as $\mathcal{C}^S := \{i : u_i < 0\} \equiv \{1, \cdots, n\}$. The coalition of reform supporters is denoted by $\mathcal{C}^R := \{i : u_i \geq 0\} \equiv \{n + 1, \cdots, I\}$.[11] From a utilitarian standpoint, we assume that

[11] Breaking the tie $u_i = 0$ by favoring the status quo instead of the reform or by randomizing the

adopting the reform is efficient while rejecting it is deemed inefficient:

$$\sum_{i\in\mathbb{I}} u_i > 0. \tag{2}$$

Stage 1*: The transfer promises game.* In the first stage game, each player i announces a transfer function defined as a vector of functions $t_i = (t_{i1}, ..., t_{iI})$, where $t_{ij} : \mathcal{O} \to \mathbb{R}^+$ represents the non-negative transfer function that player i is promising to player j conditional on the vote outcome, with $t_{ii} = 0$. Transfer promises are assumed to be credible, enforceable, and irrevocable. That is, any transfer promises contingent on a specific voting outcome *must* be honored by the initiator of the promises, the "promisers," in favor of the recipients, the "promisees," if that outcome is realized. Importantly, promises of transfers aimed at a player cannot be refused by that player. However, any transfer recipient can neutralize that transfer by making a transfer in the reverse direction to the promiser.

We denote the set of admissible transfer promises $\mathcal{A}$ and the set of admissible transfer promises profiles $\mathcal{A}^I$.[12] Let t_i and $t = (t_1, ..., t_I)$ denote generic elements of $\mathcal{A}$ and $\mathcal{A}^I$, respectively. We denote the set of one-sided transfer promises contingent on adopting the reform as $\mathcal{A}_R$, defined by $\mathcal{A}_R = \{t_i \in \mathcal{A} |\ t_i(S) = 0\}$. We use $\mathcal{A}_R^I$ to refer to the set of one-sided promises profiles. Let $t^0 = (t_1^0, ..., t_I^0)$ represent the degenerate transfer defined by $t_{ij}^0(O) = 0$ for all $O \in \mathcal{O}$ and for all i, j.

Given a transfer $t \in \mathcal{A}^I$, the *net transfer* r_i^t (resp. s_i^t) is the sum of all transfers promised to member i by other committee members minus the transfers that member i has promised to others contingent passing (resp. defeating) the reform. We have

$$r_i^t := \sum_j t_{ji}(R) - \sum_j t_{ij}(R), \quad s_i^t := \sum_j t_{ji}(S) - \sum_j t_{ij}(S).$$

When $r_i^t > 0$, committee member i is a net promisee, and she gets a net utility increase of r_i^t when the reform is adopted by the committee. When $r_i^t < 0$, committee member

committee's choice between the two policies does not change the main conclusions of our analysis.

[12]The set $\mathcal{A}$ is isomorphic to the set $\mathbb{R}_+^I \times \mathbb{R}_+^I$ because each transfer function within it comprises a vector of transfers contingent upon reform adoption and another vector contingent upon reform defeat.

i is a net promiser, and her utility decreases by $|r_i^t|$ when the reform is adopted by the committee. Symmetric results hold for the promises contingent on reform defeat s_i^t. Observe that, for any $t \in \mathcal{A}^I$, the net promises profiles $r^t = (r_1^t, \cdots, r_I^t)$ and $s^t = (s_1^t, \cdots, s_I^t)$ must belong to the budget set $\mathcal{P} := \{x \in \mathbb{R}^I \mid \sum_i x_i = 0\}$.

Transfer promises alter the incentives to vote for the reform, but voters retain their voting rights in the second stage game. If the promise profile t is in place and the voting profile v is played, then the payoff to player i is given by

$$\pi_i(v,t) := \begin{cases} u_i + r_i^t & \text{if} \quad \sum_i v_i \geq \kappa; \\ s_i^t, & \text{otherwise} . \end{cases} \tag{3}$$

3 Definitions of the equilibria

Our definition of equilibrium is based on subgame perfection of the overall two stages game. It is natural to define it by backward induction. We define both Nash equilibria in the absence of coordination between players and strong Nash equilibria when coalitions can form and coordinate.

In the second stage voting game, players take the transfers as given and confront a binary collective decision (R vs S). We consider sincere voting as a natural equilibrium of that game.[13] Fixing the transfer $t \in \mathcal{A}^I$ and denoting the indicator function $\mathbf{1}$ equal to one when a condition holds and zero otherwise, the equilibrium voting strategy of player i in the second stage game and the corresponding vote outcome are

$$v_i(t) = \mathbf{1}_{u_i + r_i^t \geq s_i^t}, \qquad O(t) = R\ \mathbf{1}_{\sum_i v_i(t) \geq \kappa} + S\ \mathbf{1}_{\sum_i v_i(t) < \kappa}. \tag{4}$$

We now define the equilibrium in the overall two stages game associated to the transfer $t \in \mathcal{A}^I$. Using sub-game perfection, the payoff profile is denoted by $\widehat{\pi}(t) =$

[13] The median voter theorem (Black [1958]) applies because the heterogeneity is single dimensional and the vote is binary. As a result, voting sincerely whereby each player votes to maximize personal preferences without strategic considerations constitutes an undominated Nash equilibrium in this setting.

$(\widehat{\pi}_1(t), \cdots, \widehat{\pi}_I(t))$, where $\widehat{\pi}_i(t) = \pi_i(v(t), t)$. We recall that $v(t) = (v_1(t), \cdots, v_I(t))$ where $v_i(t)$ is defined in equation (4) for each i. Therefore, the first stage game can be seen as a one period game with payoffs profile function $\widehat{\pi} : \mathcal{A}^I \to \mathbb{R}^I$. We refer to $\widehat{\pi}_i(t)$ as the *post-transfer utility* of member i.

Our equilibrium definition applies under conditions where players cannot punish each other with negative transfers and where promised transfers cannot be revoked. Once promised, players cannot retract transfers but can only deviate by raising the promised transfers relative to the existing promised transfers. With this assumption in mind, we now define sub-game equilibria and strong sub-game equilibria.

A transfer $t \in \mathcal{A}^I$ is a *sub-game perfect Nash equilibrum* or simply Nash equilibrium if no single player i can unilaterally deviate by promising additional transfers $\tilde{t}_i \in \mathcal{A}$ and improve her payoff, that is,

$$\widehat{\pi}_i(t) < \widehat{\pi}_i(t_i + \tilde{t}_i, t_{-i}),$$

where the transfer promises t_{-i} denotes the individual transfers of all players except i, and where $(t_i + \tilde{t}_i, t_{-i})$ denotes the transfer function $(t_1, \cdots, t_{i-1},\ t_i + \tilde{t}_i,\ t_{i+1}, \cdots, t_I) \in \mathcal{A}^I$. We denote by $\boldsymbol{\mathcal{N}}$ the set of Nash equilibria.

To define strong equilibria, we first define *the blocking coalitions*. We say that a coalition of committee members $\mathcal{C} \subset \mathbb{I}$ *blocks* the transfer function $t \in \mathcal{A}^I$ if there exists an (incremental) transfer $\tilde{t} \in \mathcal{A}^I$ such that (i) only the players from the coalition $\mathcal{C}$ are allowed to launch the incremental transfers; that is, $\tilde{t}_i \neq 0$ if and only if $i \in \mathcal{C}$, and (ii) the deviation benefits all members of $\mathcal{C}$; $\widehat{\pi}_i(t) < \widehat{\pi}_i(t + \tilde{t})$ for all $i \in \mathcal{C}$. If the conditions (i) and (ii) are satisfied, then $\mathcal{C}$ is a *blocking coalition* for the transfer function t.[14] Notice that an implication of this definition is that if a blocking coalition exists, it must overturn the committee's decision.[15]

[14]Note that the recipients of transfers from the deviation $\tilde{t}$ are not considered part of the blocking coalition if they do not promise any transfer to other players. This aligns with the assumption that players cannot directly reject a transfer promised by another player.

[15] Overturning the committee's decision is a result of the definition of blocking coalitions, not a

A *strong sub-game perfect Nash equilibrium* is a transfer function $t \in \mathcal{A}$ such that there exists no coalition $\mathcal{C}$ that can block it. We will qualify these equilibria as strong equilibria and denote their set by $\boldsymbol{\mathcal{S}}$. Notice that every strong equilibrium is also an (Nash) equilibrium and, therefore, $\boldsymbol{\mathcal{S}} \subseteq \boldsymbol{\mathcal{N}}$. This relationship can be seen by considering a deviation of the I singleton coalitions.

4 Equilibrium characterization

We begin by examining equilibria within the set $\boldsymbol{\mathcal{N}}$. For any transfer function $t \in \mathcal{A}^I$, let $u_i^t := u_i + r_i^t - s_i^t$ be referred to as *the post-transfer net utilities.* After the transfers, we assume that m players oppose the reform, and we order the post-transfer net utilities with the permutation $\sigma : \mathbb{I} \longrightarrow \mathbb{I}$ as follows:

$$u_{\sigma_1}^t \leq \cdots \leq u_{\sigma_m}^t < 0 \leq u_{\sigma_{m+1}}^t \leq \cdots \leq u_{\sigma_I}^t. \tag{5}$$

Therefore, $O(t) = R$ if and only if $m < \hat{\kappa}$. Using these notations, we characterize the set $\mathcal{N}$ in the next proposition.

Proposition 1. *[**Characterization of Nash equilibria**] A transfer function t is an equilibrium, $t \in \boldsymbol{\mathcal{N}}$, if and only if one of the following two conditions holds:*

(i) The transfers induce the reform to be adopted ($m < \hat{\kappa}$) and $-u_{\sigma_1}^t \leq \sum_{i=m+1}^{\hat{\kappa}} u_{\sigma_i}^t$.

---

defining characteristic of them. To see this, consider a blocking coalition $\mathcal{C}$ that deviates with the incremental transfers $\tilde{t}$ so that $\widehat{\pi}_i(t+\tilde{t}) > \widehat{\pi}_i(t)$ for all $i \in \mathcal{C}$. Assume further that the committee decision is not changed by the coalition, $O(t) = O(t+\tilde{t}) = R$, for instance. In that case, $\widehat{\pi}_i(t) = u_i + r_i^t$ and $\widehat{\pi}_i(t+\tilde{t}) = u_i + r_i^t + r_i^{\tilde{t}}$ for all $i \in \mathcal{C}$. Therefore $0 < r_i^{\tilde{t}}$ for all $i \in \mathcal{C}$, which implies $0 < \sum_{i \in \mathcal{C}} r_i^{\tilde{t}}$. Since all members of the blocking coalition are transfer promisers, $\tilde{t}_i \neq 0$ if and only if $i \in \mathcal{C}$, we have

$$\sum_{i \in \mathcal{C}} r_i^{\tilde{t}} = \sum_{i \in \mathcal{C}} \left(\sum_{j \in \mathbb{I}} \tilde{t}_{ji}(R) - \sum_{j \in \mathbb{I}} \tilde{t}_{ij}(R) \right) = \sum_{i \in \mathcal{C}} \left(\sum_{j \in \mathcal{C}} \tilde{t}_{ji}(R) - \sum_{j \in \mathbb{I}} \tilde{t}_{ij}(R) \right) = -\sum_{i \in \mathcal{C}} \sum_{j \in \mathbb{I}/\mathcal{C}} \tilde{t}_{ij}(R) \leq 0,$$

where the inequality is implied by the fact that transfers are non-negative. This contradicts the assertion that $0 < \sum_{i \in \mathcal{C}} r_i^{\tilde{t}}$. A similar reasoning by contradiction applies to the case where $O(t) = O(t+\tilde{t}) = S$.

(ii) The transfers induce the reform to be defeated ($m \geq \hat{\kappa}$) and $u^t_{\sigma_I} \leq \sum_{i=\hat{\kappa}}^{m}(-u^t_{\sigma_i})$.

The first type of equilibria presented in Proposition 1 occurs when $m < \widehat{\kappa}$ and results in reform adoption. It is characterized in condition (i) of Proposition 1. If a player deviates from t by offering additional transfers to others, her payoff can only improve if the reform is defeated. If the group decision remains unchanged, the deviating player's payoff should be lower than before the deviation because she has transferred resources to others. Hence, the deviations can only be initiated by reform opponents who aim to sway additional players to vote against the reform. The member with the strongest incentives to make such a deviation is player σ_1 with the lowest post-transfer net utility $u^t_{\sigma_1}$. The pivotal coalition of reform supporters that is "cheapest" to entice to vote against the reform and sway the committee into rejecting it is formed by the members $\sigma_{m+1}, \cdots, \sigma_{\widehat{\kappa}}$. The inequality in condition (i) of Proposition 1 says that a unilateral deviation to persuade players $\sigma_{m+1}, \cdots, \sigma_{\widehat{\kappa}}$ to vote against the reform is not profitable for player σ_1. The characterization in condition (ii) of Proposition 1 is supported by symmetrical intuitions as those underlying condition (i).

The restrictions in Proposition 1 also suggest the existence of multiple equilibria,[16] which do not always lead to the adoption of the reform. If many supporters have small stakes in the reform while only a few opponents have large stakes, unilateral Nash deviations by reform supporters can be suboptimal due to the prohibitively high cost of persuading the opponents. The following example illustrates this scenario.

Example 1. *Consider a three-member committee where decisions are taken by unanimity: $\kappa = 3$ and $\widehat{\kappa} = 1$. Utilities are given by $u = (u_1, u_2, u_3) = (-4,\ 2,\ 3)$. The reform is socially optimal since $\sum_i u_i = 1 > 0$. In the absence of transfers, the reform is defeated because committee member 1 vetoes it. The degenerate transfer $t^0 = 0$ is however an equilibrium, $t^0 \in \mathcal{N}$. Deviations from t^0 by either member 2 or member 3 necessitate a transfer of more than 4 units to member 1 to dissuade her from vetoing*

[16]We will show in Proposition 2 that the set of strong equilibria, $\boldsymbol{\mathcal{S}}$, is indeterminate. Consequently, the larger set $\boldsymbol{\mathcal{N}}$ is also indeterminate as $\boldsymbol{\mathcal{S}} \subseteq \boldsymbol{\mathcal{N}}$.

the reform. Such deviations are suboptimal, since any unilateral transfer exceeding 4 units by either member 2 or member 3 results in a negative payoff, reducing their posy-transfer utilities compared to not initiating any transfer. Hence $t^0 \in \boldsymbol{\mathcal{N}}$.

We now provide a characterization of the set of strong equilibria.

Proposition 2. *[**Characterization of strong equilibria**] The set of strong equilibria $\boldsymbol{\mathcal{S}}$ is non-empty and efficient, $O(t) = R$ for all $t \in \boldsymbol{\mathcal{S}}$. Moreover,*

$$\boldsymbol{\mathcal{S}} = \{t \in \mathcal{A}^I \mid \sum_{\mathcal{C}} u_i + r_i^t \geq s_i^t \text{ for all coalitions } |\mathcal{C}| \geq \hat{\kappa}\}. \tag{6}$$

It is well-known that the conditions for Strong Nash equilibria are highly stringent. Proposition 2 demonstrates that Strong Nash equilibria exist within our context. This existence result is novel and provides a rare instance of a game where such equilibria actually exist. This proposition also reveals that Strong Nash equilibria are both indeterminate and efficient, as they facilitate the approval of the reform. The successful coalitional cooperation mitigates the "majority coercion" political failure inherent in the binary voting system. By introducing transfers before voting, reform supporters can modulate their transfer promises, influencing opponents by leveraging transfers that are contingent on the reform's approval. The Coasean intuition is, therefore, valid in our game. As we will discuss in Section 7.2, the assumption of no retraction on promises as well as the assumption that promises are contingent on vote outcome are crucial for the result.

The characterization in (6) ensures that in a strong equilibrium, once the transfers are implemented, the reform remains socially optimal for any coalition that has the power to overturn it. Strong equilibria are defined by the system of linear inequalities (6) on net transfers, forming a convex polyhedron within the set $(r, s) \in \mathcal{P}^2$. This framework enables a detailed analysis of strong equilibria.[17] The system of inequalities

[17]For example, notice that the inequalities (6) hold for $\widehat{\kappa}$, and they will also hold for any quota $\widehat{\kappa}'$ such that $\widehat{\kappa}' \geq \widehat{\kappa}$. This means that, for any quota κ, the set of strong equilibria under the quota rule κ is a subset of the set of strong equilibria under quota rule $\kappa - 1$. In particular, the set of strong equilibria under the unanimity rule is a subset of the set of strong equilibria under the majority rule.

(6) restricts the transfer functions t solely through the net transfers (r^t, s^t). The system has in general multiple solutions suggesting the indeterminacy of the strong equilibria. Observe that if the net promises $(r,s) \in \mathcal{P}^2$ satisfy the inequalities (6) then, by linearity, the one-sided net promises $(\widehat{r} = r - s, \widehat{s} = 0)$ also satisfies those inequalities. Hence any one-sided transfer function $\hat{t} \in \mathcal{A}_R^I$ satisfying $r^{\hat{t}} = \widehat{r}$ is also a strong equilibrium. This means that one-sided transfer promises contingent on reform adoption are sufficient to reach efficiency. We discuss an example to illustrate the results of Proposition 2.

Example 2. *We revisit Example 1 of a committee with three members operating under unanimity with $u = (-4, 2, 3)$. We recall that the degenerate transfer t^0 is a Nash equilibrium. Proposition 2 implies that the transfer t^0 is not a strong equilibrium because $u_1 = -4 < 0$ and $\widehat{\kappa} = 1$. This occurs because members 2 and 3 can create a blocking coalition of the transfer t^0 by compensating member 1 to support the reform, thereby enabling the committee to enact it.*

Consider now the one-sided transfer t defined by $t_{21}(R) = 1$, $t_{31}(R) = 3$ and zero otherwise. The associated net transfers are given by $(r^t = (4, -1, -3), s^t = 0)$, the resulting payoff is $\widehat{\pi}(t) = (0, 1, 0)$ and hence, $O(t) = R$. Moreover, the transfer t defined above constitutes a strong equilibrium, as it satisfies the inequalities (6).

To illustrate the difference between strong equilibria and Nash equilibria in the context of this example, Panel A of Figure 1 depicts the set of Nash equilibria $\boldsymbol{\mathcal{N}}$ and the set of strong equilibria $\boldsymbol{\mathcal{S}}$. We plot in Panel A the net transfer r_3 on the y-axis versus the net transfer r_1 on the x-axis, assuming the transfers are one-sided, $t(S) = 0$. The inequalities (6) show that a one-sided transfer t is a strong equilibrium if and only if $r = r^t$ satisfies:

$$-4 + r_1 \geq 0, \quad 2 + r_2 \geq 0, \quad 3 + r_3 \geq 0, \quad r_1 + r_2 + r_3 = 0.$$

As shown in Panel A of Figure 1, the light blue triangle represents the set of strong equilibria. For the Nash equilibria, however, besides the strong equilibria where all post-transfer utilities are nonnegative, we may allow one member has negative post-transfer *utility. The resulting transfer does not meet the strong Nash requirement but can still qualify as a Nash equilibrium, provided that no single player can benefit by deviating to entice any member into supporting the reform. For example, when member* 1 *has negative post-transfer utility, that is,* $\sigma_1 = 1$ *in Example 1, then, by Proposition 1 (ii) with* $m = \hat{\kappa} = 1$*, we have*

$$2 + r_2 \leq -(-4 + r_1), \quad 3 + r_3 \leq -(-4 + r_1), \quad r_1 + r_2 + r_3 = 0.$$

This set is shown as a dark blue triangle in the upper left corner of Panel A in Figure 1. The set is partially visible due to figure truncation, with the true set forming a cone extending from the triangle. Similarly, Nash equilibria in which member 2 *or* 3 *has negative post-transfer utility are represented by dark blue regions in the upper right and lower right corners of Panel A. Notably, the reform is defeated in all of these dark blue regions (Nash equilibria), while it is enacted only within the light blue triangle (strong equilibria) in Panel A.*

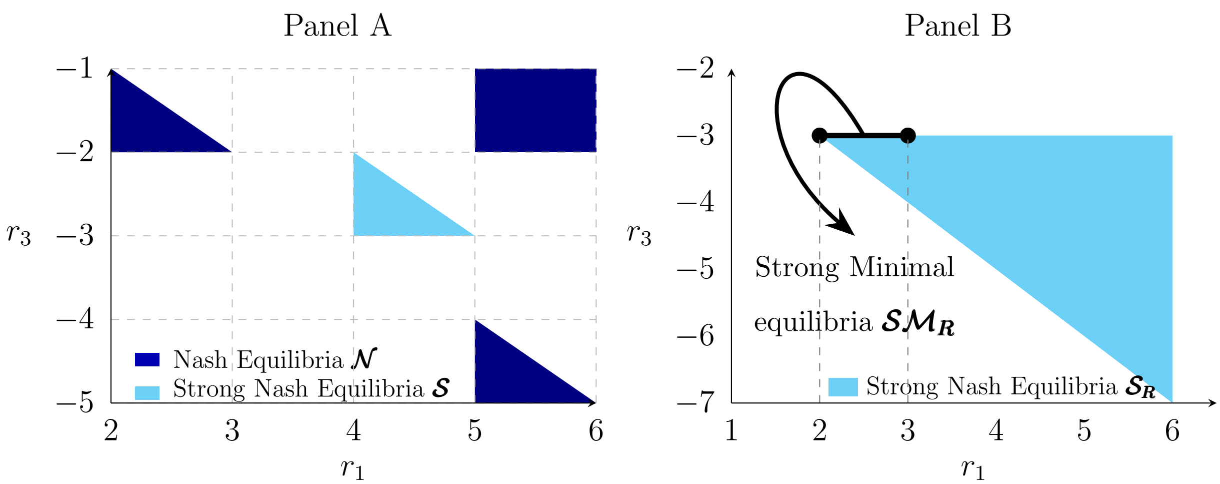

Figure 1: The two panels describe the set of equilibria for a three members committee in a the plane (r_1, r_3). We consider in Panel A the case $u = (-4,\ 2,\ 3)$ of a committee operating under the unanimity rule (covered in Examples 1 and 2). In Panel B, we consider the case $u = (-4, 1, 5)$ of a committee operating under majority rule covered in Example 3. In both panels, we assume that the transfer are one sided: $t(S) = 0$.

In the remainder of the paper, we focus on strong equilibria, given that our study involves committees where members can coordinate their actions. This approach offers new theoretical insights, as applying strong equilibrium analysis to evaluate vote trading represents a novel contribution to the literature to our knowledge. Since each strong equilibrium can be linked to a one-sided strong equilibrium transfer, we will focus on one-sided transfer promises contingent on passing the reform and express the constraints of strong equilibria in terms of the net transfers $r \in \mathcal{P}$.[18] We then consider the set of strong equilibria that minimizes total promised transfers, resulting in the set of strong one-sided equilibria that achieves this minimization. The next section will formally define this set, demonstrate its existence, and highlight the general properties of strong minimal equilibria. The minimality assumption will aid in predicting the types of transfers associated with strong equilibria in the subsequent analysis.

5 Strong equilibria with minimal transfer (SM)

We now define the set of *Strong Minimal* (SM) net transfers $r \in \mathcal{P}$ denoted by $\boldsymbol{\mathcal{SM}}_R$. We first denote by $\boldsymbol{\mathcal{S}}_R$ the set of strong equilibrium net transfers, defined as the set of net promises that can be supported by a one-sided strong equilibrium transfer:

$$\boldsymbol{\mathcal{S}}_R = \{r \in \mathcal{P} \ \mid \ \exists t \in \mathcal{A}_R^I \text{ such that } t \in \boldsymbol{\mathcal{S}} \text{ and } r^t = r\}.$$

[18]Since the equilibrium restrictions are defined by the difference between net transfers contingent on the adoption of the reform and those contingent on its defeat, the impact of strong equilibria is reflected in these differences. Therefore, given that utility is linear in net transfers, it is appropriate to focus the analysis on net transfers associated with the reform's adoption. For net transfer, $r \in \mathcal{P}$ there exist multiple transfer promises $t \in \mathcal{A}_R^I$ such that $r^t = r$. To see this, consider the linear system $r_i^t := \sum_j t_{ji}(R) - \sum_j t_{ij}(R)$ with I equations and $I^2 - I$ unknown consisting of $t_{ij}(R)$. Given the large number of unknowns, the system admits multiple solutions. Consider any solution $t_{ij}(R)$ of that system and observe that the one-sided transfer profile $t_{ij}(R) + C$ is also a solution of the system of equations for any constant $C > 0$. Then, for C sufficiently large, we have $t_{ij}(R) + C \geq 0$ and thus $\{t_{ij}(R) + C\}_{i,j} \in \mathcal{A}_R^I$ is a one-sided transfer promises that produce the net promise r.

For any $r \in \mathcal{P}$, the total transfers associated to r is defined as

$$\mathcal{T}_r := \frac{1}{2} \sum_{\mathbb{I}} |r_i|. \tag{7}$$

A net promises profile $r \in \mathcal{P}$ contingent on the reform is a Strong Minimal equilibrium or "SM" if (i) it is strong equilibrium, $r \in \boldsymbol{\mathcal{S}}_R$, and (ii) it minimizes the total promises transfer among all strong equilibria,

$$\boldsymbol{\mathcal{SM}}_R := \{r \in \boldsymbol{\mathcal{S}}_R \mid \mathcal{T}_r = \inf_{r' \in \boldsymbol{\mathcal{S}}_R} \mathcal{T}_{\boldsymbol{r'}}\}.$$

The following proposition shows that the Strong Minimal equilibria exist.

Proposition 3 (Existence). *The set of SM equilibria net transfers $\boldsymbol{\mathcal{SM}}_R$ is a non-empty convex and compact subset of $\mathcal{P}$.*

Relative to the set $\boldsymbol{\mathcal{S}}_R$ of Strong equilibria, the set of SM equilibria $\boldsymbol{\mathcal{SM}}_R$ adds the restriction that the equilibria are achieved in the "cheapest possible way." The set $\boldsymbol{\mathcal{SM}}_R$ is a subset of the set $\boldsymbol{\mathcal{S}}_R$ as the following example illustrates.

Example 3. *Consider a committee consisting of 3 members, ruled by the majority rule, $\kappa = \widehat{\kappa} = 2$, and with utilities $u = (-4, 1, 5)$. Inequality (6) shows that the net promise transfer $r \in \mathcal{P}$ is a strong equilibrium, that is, $r \in \boldsymbol{\mathcal{S}}_R$, if and only if:*

$$r_1 + r_2 \geq 3, \quad r_1 + r_3 \geq -1, \quad r_2 + r_3 \geq -6, \quad r_1 + r_2 + r_3 = 0.$$

The set $\boldsymbol{\mathcal{S}}_R$ is depicted as the light blue triangle in Panel B of Figure 1. Moreover, for any $r \in \boldsymbol{\mathcal{S}}_R$, $\mathcal{T}_r = \frac{1}{2}(|r_1| + |r_1 + r_3| + |r_3|) \geq |r_3| \geq 3$, and the inequality holds as an equality if and only if $r_3 = -3$ and $2 \leq r_1 \leq 3$. Thus, the set of strong minimal equilibria $\boldsymbol{\mathcal{SM}}_R$, is represented by the horizontal side of the triangle bounded by the points $(2, -3)$ and $(3, -3)$, with $\mathcal{T}_r = 3$.

Example 3 illustrates that a multiplicity of strong equilibrium net transfer persists even when we focus on the transfer promises that minimize the total transfers (7).

However, by examining these strong minimal equilibria, we can uncover general properties of the net transfers. The next proposition shows that there exists *a critical voter*, defined as a reform supporter $k_* \in \mathcal{C}^R$ such that all promisers of transfers belong to the coalition $\{k_*, \cdots, I\}$, and all transfer recipients belong to the coalition $\{1, \cdots, k_* - 1\}$.

Proposition 4. *[**Top-down equalizing transfers**] There exists a critical voter $k_* \in \mathcal{C}^R$ such that any SM equilibrium net transfers profile $r \in \boldsymbol{\mathcal{SM}}_R \backslash \{0\}$, satisfies*

$$-u_j \leq r_j \leq 0 \leq r_i, \quad \text{and} \quad u_i + r_i \leq u_j + r_j \quad \text{for all} \quad i < k_* \leq j, \tag{8}$$

and the total transfers associated to the SM equilibrium r satisfies:

$$\mathcal{T}_r = \sum_{i<k_*} r_i = \sum_{j \geq k_*} (-r_j). \tag{9}$$

Proposition 4 demonstrates that, in any SM equilibrium, only the reform supporters whose utility weakly exceeds that of a critical voter k_* can promise a transfer. Additionally, these transfer promisers are constrained by the individual rationality condition, $-u_j \leq r_j$ for $j \geq k_*$, which prevents them from transferring more than the utility they gain from the reform. The individual rationality condition is guaranteed by minimality.[19]

Proposition 4 further reveals that transfer recipients must belong to the coalition $\{1, \cdots, k_* - 1\}$. When this coalition includes some reform supporters, the recipients of transfers may be either opponents or supporters of the reform. In the latter case, reform supporters with higher utility promise transfers to other reform supporters with lower utility. The utility rank of the critical voter κ_* among other committee members is influenced by the voting rule and the distribution of utilities. The final

[19] In example 3, the net transfer $r = (6, 1 - 7)$ is a strong equilibrium resulting in payoff $\hat{\pi} = (2,\ 2,\ -2)$. In that equilibrium, member 3 transfers 7, an amount that is larger than her maximal utility from the game, $u_3 = 5$. These "excessive" transfers are consistent with the strong equilibrum equilibrium since, once they are in place, players who initiate them cannot renege on them. However, excessive transfers are eliminated with the minimality requirement.

SM equilibrium restriction, given by the middle inequality in (8), requires that any member of the promisers' coalition $\{k_*, \cdots, I\}$ must have a post-transfer utility no lower than that of any member of the recipients' coalition $\{1, \cdots, k_* - 1\}$. Minimal equilibria do not overturn the ranking of utilities between promisers and promisees.

Because of the endogeneity of the critical voter k_*, Proposition 4 remains agnostic regarding both the magnitude of transfer promises and the identities of the committee members involved in these transfers. In particular, the promise recipients can be reform supporters or reform opponents. The total transfers depend on the distribution of utilities u_i and the majority threshold κ. In the next section, we provide more detailed descriptions of the SM equilibria.

6 Strong Nash Minimal equilibria implications

In this section, we examine both the magnitude and direction of SM equilibrium transfer flows and explore how multiplicity arises in various scenarios. We first examine the case where the reform is defeated without transfers, and then discuss the scenario where the reform passes without transfers. The discussion and presentation of the principles are illustrated using specific numerical examples. However, these principles are general, and a detailed exposition of the general results, along with their proofs, can be found in the Supplemental Appendix SA. Before proceeding, we define the aggregate utility (resp. dis-utility) of the reform supporter (resp. opponents) by

$$U_R := \sum_{\mathcal{C}^R} |u_i| \equiv \sum_{i=n+1}^{I} u_i, \quad U_S := \sum_{\mathcal{C}^S} |u_i| \equiv \sum_{i=1}^{n} (-u_i). \tag{10}$$

6.1 Equilibrium implications when the reform is defeated

We examine scenarios where the reform lacks sufficient support without transfers, specifically when $| \mathcal{C}^R | < \kappa$ and $O(t^0) = S$. In this context, the critical voter from Proposition 4 is identified as $\kappa_* = n + 1$. Thus, only the opponents of the reform can receive transfers. The minimum total transfer is given by $\mathcal{T}_r = U_S$, indicating that reform supporters must compensate reform opponents sufficiently to cover their total utility loss from the adoption of the reform. Additionally, Proposition 4 demonstrates that, following these transfers, the utility of any reform opponent cannot exceed that of any reform supporter. In general, there are multiple SM equilibria because there are various ways to allocate the cost of transfers among reform supporters and different methods for distributing the transfers among reform opponents. However, when reform supporters are at least two members short of reaching the quota κ, all equilibria involve compensating each reform opponent sufficiently to make each one of them indifferent between the reform and the status quo, $r_i = -u_i$ for all $i \in \mathcal{C}^S$. To illustrate this subcase, consider an example.

Example 4. *Consider a committee consisting of three members, with utilities* $u = (-2, -1, 10)$. *We have* $\mathcal{C}^R = \{3\}$, $\mathcal{C}^S = \{1, 2\}$, *and* $U_S = 3$. *In an SM equilibrium, member* 3 *must transfer* 3 *units of utility to members* 1 *and* 2, *regardless of whether the committee operates on a majority or unanimity basis.*

When the committee operates under the unanimity rule, the coalition $\mathcal{C}^R$ *is two members short of being decisive. In this case, all SM transfer promises* t *yield the same net transfer* $r = (2, 1, -3)$ *and the same post-transfer utilities* $\hat{\pi}(t) = (0, 0, 7)$.

When the committee operates under a majority rule: $\kappa = \widehat{\kappa} = 2$, *there are multiple SM equilibria because the coalition* $\mathcal{C}^R$ *is one member short of reaching the quota* 2. *The transfer of* 3 *units of utility initiated by member* 3 *can be arbitrarily distributed among member* 1 *and* 2. *For example the net transfers* $r = (3, 0, -3)$ *and* $r = (0, 3, -3)$ *are both SM equilibria.*

6.2 Equilibrium implications when the reform is adopted

We now assume that the reform has sufficient support to be enacted without transfers, $O(t^0) = R$ or equivalently that the number of reform supporters is large, $| \mathcal{C}^R | \geq \kappa$. Members of the coalition $\mathcal{C}^S$ can promise transfers contingent on defeating the reform to some members of the coalition $\mathcal{C}^R$ to increase the support for their preferred policy S

and lead the committee to defeat the reform. The members of the coalition $\mathcal{C}^R$ who are more susceptible to being enticed to vote for the status quo are those with the lowest utilities in the reform. When the enticements are persuasive, some reform supporters are converted to reform opponents, and these conversions represent a pivotal event if a blocking coalition exists. We denote the coalition of these reform supporters by

$$\underline{\mathcal{C}}^R := \{n+1, .., \widehat{\kappa}\}. \tag{11}$$

Since $\mathcal{C}^S \cup \underline{\mathcal{C}}^R = \{1, .., \widehat{\kappa}\}$, members of $\underline{\mathcal{C}}^R$ are numerous enough to defeat the reform if they vote against it with the reform opponents. Denote by $\underline{U}^R := \sum_{\underline{\mathcal{C}}^R} u_i \equiv \sum_{i=n+1}^{\widehat{\kappa}} u_i$ the aggregate utility for the reform of the coalition $\underline{\mathcal{C}}^R$.

For the members of the coalition $\mathcal{C}^S$, the incentive to entice members of the coalition $\underline{\mathcal{C}}^R$ to vote for the alternative S arises only when there are gains from trade. Specifically, by switching from the decision R to the decision S, members of the coalition $\mathcal{C}^S$ gain the aggregate amount U^S. To entice members of the coalition $\underline{\mathcal{C}}^R$ to vote for S, members of the coalition $\mathcal{C}^S$ incur the aggregate utility cost $\underline{U}^R$. Therefore, we define the aggregate gains from trade of the members of the coalition $\mathcal{C}^S$ as

$$G^S := U^S - \underline{U}^R = - \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i \equiv - \sum_{i=1}^{\widehat{\kappa}} u_i. \tag{12}$$

The sign of G^S is the key factor in characterizing the equilibrium transfer.

When the gain from trade G^S is non-positive, the only SM equilibrium is the degenerate transfer t^0 since the reform opponents have no incentives to entice the members of the coalition $\underline{\mathcal{C}}^R$ to vote against the reform.

When $G^S > 0$, transfers are necessary to prevent coalitions of reform opponents from blocking the transfer t^0, enticing some reform supporters into opposing the reform and coercing the remaining reform supporter.

Assuming $|\, \mathcal{C}^R \,| \geq \kappa$ and $G^S > 0$, two cases can occur. To describe them, we define the aggregate surplus of utility of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ relative to member $\widehat{k} \in \underline{\mathcal{C}}^R$ by

$$\Delta U_{\widehat{\kappa}} := \sum_{j \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} [u_j - u_{\widehat{\kappa}}] \equiv \sum_{j=\widehat{\kappa}+1}^{I} [u_j - u_{\widehat{\kappa}}]. \tag{13}$$

The variable $\Delta U_{\widehat{\kappa}}$ represents the maximum aggregate transfer that members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ can promise while keeping their after transfer utilities above that of member $\widehat{\kappa}$.

6.2.1 Case 1: First order preemption.

We assume that $\mid \mathcal{C}^R \mid \geq \kappa$, $G^S > 0$ and $\Delta U_{\widehat{\kappa}} \geq G^S$. When $\Delta U_{\widehat{\kappa}} \geq G^S$, the members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ can afford to promise a total transfer of G^S while maintaining their post-transfer utilities above that of all the members of the coalition of promises $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. In this case, members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ can be compensated for the forgone gain realized by forming a blocking coalition, i.e., G^S, without creating new targets for enticement against the reform. The critical voter is thus $k_* = \widehat{\kappa} + 1$ and all equilibria involve a total transfer of $\mathcal{T}_r = G^S$ from the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R = \{\widehat{\kappa} + 1, \cdots, I\}$ to the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R = \{1, \cdots, \widehat{\kappa}\}$. In all SM equilibria, the post-transfer individual utilities remain larger for all members of $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ than that of any member of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. To illustrate this case, we examine an example.

Example 5. *Consider a committee with 4 members governed by the majority rule $\kappa = 3$ and utilities $u = (-5,\ 1,\ 2,\ 10)$. In this case, $\widehat{\kappa} = 2$, $\mathcal{C}^S = \{1\}$, $\mathcal{C}^R = \{2,\ 3,\ 4\}$, $\underline{\mathcal{C}}^R = \{2\}$, $\mathcal{C}^R/\underline{\mathcal{C}}^R = \{3,\ 4\}$, and $G^S = 4$. The aggregate surplus of utility of the coalition $\{3, 4\}$ is $\Delta U_2 = (2 - 1) + (10 - 1) = 10 > G^S = 4$. Therefore the coalition $\{3, 4\}$ can afford to promise a transfer of 4 without violating the ordering condition of the post-transfer utilities from Proposition 4. In any SM equilibrium, the coalition $\mathcal{C}^R/\underline{\mathcal{C}}^R = \{3, 4\}$ should promise a total transfer of $G^S = 4$ to the coalition $\{1, 2\}$. The critical vote trader from Proposition 4 is thus $k_* = 3$. There are multiple SM equilibria. For example, the net transfers $r = (3,\ 1,\ 0,\ -4)$ is an SM equilibrium. It produces the post-transfer utilities $\widehat{\pi}(r) = (-2,\ 2,\ 2,\ 6)$ and includes a "circle the wagon" transfer, as it involves a transfer from reform supporter 4 to another reform supporter, member 2, who has a weaker utility. The net transfer $r' = (4,\ 0,\ 0,\ -4)$ is also an SM equilibrium as it produces the post-transfer utilities $\widehat{\pi}(r') = (-1,\ 1,\ 2,\ 6)$.*

The net transfer $r'' = (2,\ 2, -1, -3)$ *achieves the minimum total transfer promises* $\mathcal{T}_{r''} = 4$ *but is not an SM equilibrium. This is because the post-transfer utilities* $\widehat{\pi}(r'') = (-3,\ 3,\ 1,\ 7)$ *violate the ordering condition* $\widehat{\pi}_2(r'') \leq \widehat{\pi}_3(r'')$*. Since* $\widehat{\pi}_1(r'') + \widehat{\pi}_3(r'') = -2 < 0$ *after the net transfers* r'' *are implemented, member 3 becomes a new target for enticement by member 1. Thus the coalition* $\{1\}$ *can block the net transfer* r''*, indicating that the net transfer* r'' *is not a strong equilibrium.*

6.2.2 Case 2: Higher order preemption

We now assume that $\mid \mathcal{C}^R \mid \geq \kappa$, $G^S > 0$ but this time $\Delta U_{\widehat{\kappa}} < G^S$. When $\Delta U_{\widehat{\kappa}} < G^S$, if the members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ promise a total transfer of G^S to compensate the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ for not undermining the reform, they violate the ranking of post-transfer utilities. Consequently, some members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ become targets for new transfers aimed at enticing them to vote against the reform. These additional opportunities of transfers will require second-order compensation. For this reason, it is intuitive that the total transfer of SM equilibria must be larger than G^S. The results from the supplemental appendix SA show that the critical vote trader is a member $k_* \in \underline{\mathcal{C}}^R$ defined by

$$u_{k_*-1} \leq u_* < u_{k_*}, \quad \text{where} \quad u_* := \frac{1}{\kappa - 1} \sum_{j \in \mathbb{I}} u_j > 0, \tag{14}$$

and that the total transfer for any SM equilibrium net transfer r is given by $\mathcal{T}_r = \mathcal{T}_* := \sum_{j \geq k_*} \left[u_j - u_* \right] > G^S$. To illustrate this case, we discuss an example.

Example 6. *Consider a slight variation of Example 5, where a committee with* 4 *members is ruled by majority rule,* $\kappa = 3$*, and where the utilities are now given by* $u = (-5,\ 1,\ 2,\ 3)$*. In this case* $\widehat{\kappa} = 2$*,* $\mathcal{C}^S = \{1\}$*,* $\mathcal{C}^R = \{2,\ 3,\ 4\}$*,* $\underline{\mathcal{C}}^R = \{2\}$*,* $\mathcal{C}^R / \underline{\mathcal{C}}^R = \{3,\ 4\}$*, and* $G^S = 4$*. We have* $\Delta U_{\widehat{\kappa}} \equiv \Delta U_2 = 3 < 4 = G^S$ *and thus the coalition* $\{3, 4\}$ *cannot afford to promise the transfers of* 4 *without creating new targets of enticement. In the first step, members* 3 *and* 4 *will promise a transfer to member* 1 *in order to equalize their utilities with that of member* 2*. The net transfers* *are therefore* $r^1 = (3,\ 0,\ -1,\ -2)$*, which result in the post transfer payoff* $\widehat{\pi}(r^1) = (-2,\ 1,\ 1,\ 1)$*. The net transfer is not an SM equilibrium because it is not strong Nash:* $\widehat{\pi}_1(r^1) + \widehat{\pi}_2(r^1) = -1 < 0$*, and thus additional transfer promises need to be made. Using the symmetry of this specific example, we conjecture that for an SM equilibrium, each member of the coalition* $\{2,3,4\}$ *must transfer an amount* x *to member* 1*, which results in the net transfer* $r^2 = (3x,\ -x,\ -x,\ -x)$ *and thus the post-transfer utilities* $\widehat{\pi}(r^1 + r^2) = (-2 + 3x,\ 1 - x,\ 1 - x,\ 1 - x)$*. If we select* $x = 1$*, the net transfer* $r^1 + r^2 = (6,\ -1,\ -2,\ -3)$ *is a strong equilibrium in which total transfer* $\mathcal{T}_{r^1+r^2} = 6$ *is too large. To achieve the minimum total transfer, we solve the equation* $(-2 + 3x) + (1 - x) = 0$ *and get the solution* $x = 1/2$*. The net transfers promise* $r^1 + r^2 = (9/2,\ -1/2,\ -3/2,\ -5/2)$ *is an SM equilibrium with total transfer* $\mathcal{T}_{r^1+r^2} = 9/2 > 4$ *and payoff* $\widehat{\pi}(r^1 + r^2) = (-\frac{1}{2},\ \frac{1}{2},\ \frac{1}{2},\ \frac{1}{2})$*. The critical voter is thus* $k_* = 2$*.*

7 Discussion

In this section, we discuss some additional implications of the equilibrium transfers and the restrictions on the types of transfers we have considered. We also discuss the dynamic implementation of our equilibria.

7.1 Reaching across the aisle transfers

The discussion in Section 6 shows that when the reform lacks sufficient voting support to be enacted without the use of transfers, transfer recipients are always reform opponents and thus all transfers are reaching across the aisle in that case. In cases of weak voting support for the reform, promises involving "circle the wagon" transfers are, therefore, not expected to occur in SM equilibria.

The analysis in full generality is done in Proposition SB.1 from the Supplemental Appendix B where we provide necessary and sufficient conditions under which all equilibrium promises are of the across the aisle type. The proposition shows that when the reform has sufficient voting support without transfers, circle the wagon transfers where some transfers are directed to reform supporters with the weakest utilities exist in the set of SM equilibria. Circle the wagon transfers are excluded in the knife edge cases where a specific subset of reform supporters with the weakest utilities have equal utilities.

7.2 Alternative transfer promises

A key assumption in our definition of blocking coalitions is that members cannot retract their transfer promises. The assumption is particularly relevant for committees where reputation is very important for future interactions, making the cost of reneging on promises prohibitively high. The assumption is also crucial to get our existence result. To see this, assuming retraction is possible, we say that a coalition $\mathcal{C} \subseteq \mathbb{I}$ is a blocking coalition with retraction of the transfer function $t \in \mathcal{A}^I$ if there exists a transfer function $\tilde{t} \in \mathcal{A}^I$ such that (i) only the players from the coalition $\mathcal{C}$ are allowed to deviate from t, $\mathcal{C} = \{i \mid \tilde{t}_i \neq t_i\}$, and (ii) the deviation benefits all members of $\mathcal{C}$; $\widehat{\pi}_i(\tilde{t}) > \widehat{\pi}_i(t)$ for all $i \in \mathcal{C}$. A strong Nash equilibrium with retraction is a transfer function that is immune to any blocking coalition with retraction. Under this definition, the requirements for a strong Nash equilibrium become more stringent compared to one without retraction. In particular, if a strong equilibrium with retraction exists, then it is also a strong equilibrium with no retraction. The following proposition demonstrates that the requirements of strong Nash with retraction can become excessively demanding, potentially jeopardizing the existence of such an equilibrium.

Proposition 5. ***[Non-existence of Strong Nash equilibrium with retraction]*** *Consider the committee in Example 4, consisting of three members with utilities* $u = (-2, -1, 10)$ *and operating under majority rule,* $\kappa = 2$*. In this scenario, no strong Nash equilibrium exists if members can retract their promises.*

The no-retraction assumption in our strong Nash equilibria implies that promisers commit to a minimum level of transfers, defined by the transfers already in place. In contrast, in strong equilibria with retraction, the commitment is less stringent; players need only commit to a non-negative transfer amount, as they are allowed to reduce their promises when deviating. When punishment is possible but limited, players commit to a maximum penalty rather than a specified level of non-negative transfers. Our model precludes retraction and demands, thus, the highest level of commitment among these variations, leading to a novel existence result for strong Nash equilibria. Proposition 5 shows that if the commitment requirement is slightly relaxed by allowing retraction while maintaining non-negative transfers, strong Nash equilibria no longer exist. A corollary of the proposition is that strong equilibria with punishment do not exist in the context of the example studied in Proposition 5.[20]

In our study, transfer promises are restricted to be contingent on the voting outcome. More broadly, transfers could be based on individual votes or even entire voting profiles. Specifically, transfers contingent on the recipients' votes, often involving upfront payments, can reflect practices such as corporate vote trading. They can also model the practice of *clientelism*, where a politician (i.e., a "patron") distributes public jobs in exchange for electoral support (see, e.g., Weingrod [1968]). The analysis of numerical examples highlights a difficulty with transfers contingent on individual votes: in the second-stage game, multiple equilibria emerge, and sincere voting is not a natural equilibrium as in our current setting. When the transfers are contingent on the vote of the recipients, we do not know whether strong equilibria exist and therefore we do not know whether the Coasean intuition holds. In contrast, transfers based on vote outcomes exhibit a "contracting on contracts" feature (Katz, 2006). While the transfer promises made to a committee member are not contingent on transfers to other

[20] The non-existence of strong equilibria with punishment is perhaps not surprising in our decentralized setting. Players who are punished may retaliate, potentially leading to a cycle of revenge that hinders the existence of strong equilibria.

members, they are still influenced by these transfers, as each one affects the overall voting outcome. Intuitively, the interdependence of transfer promises contingent on vote outcome ties the continuation utility of the promisers to that of the promises recipients and supports the existence of a strong equilibrium. Our findings regarding the success of transfers contingent on vote outcomes in promoting efficiency suggest that institutions facilitating this type of promise and making retraction very costly will also promote efficiency. Our results represent thus a first step that lays the groundwork for a comprehensive analysis, providing a basis for validating the "Coasean intuition" in the context of more general form of vote trading or potentially offering evidence to the contrary.

7.3 Dynamic implementation

In our model, we assume that transfer promises are made simultaneously and do not explicitly model the game leading to equilibrium. However, once an equilibrium transfer profile is reached, it cannot be overturned, while other transfer profiles do not exhibit the same level of stability. In practice, transfers can take the form of amendments to legislative bills, involving multiple rounds of negotiation. To better understand the process through which an equilibrium is reached, it would therefore be useful to model a dynamic game of sequential decentralized transfer promises that occurs in the first stage, before the voting stage. A strong equilibrium $t \in \boldsymbol{\mathcal{S}}$ can be thought of as the summation of all the incremental promises that have been made sequentially. Starting with the transfer t^0, we can "implement" any strong equilibrium in at most two steps. In each step, the members of a blocking coalition make some transfers, reverse the committee decision and improve their individual utility relative to their utility prior to the transfers.

When the reform is defeated in the absence of transfers, $O(t^0) = S$, the equilibrium can be implemented in one step. In that step, the members of a coalition of reform supporters promise transfers to some reform opponents in a way that leaves no possibility for the formation of blocking coalitions after the transfers are promised.

In contrast, when $O(t^0) = R$, a blocking coalition in the first step must oppose the reform. To do so at the cheapest cost, its members need to offer transfers to the weakest supporters of the reform, contingent on defeating it. In a second step, members of a blocking coalition of reform supporters promise transfers in response to the first blocking coalition by first returning the transfers contingent on outcome S received in the first step and then making new promises contingent on outcome R. We formalize these implementations in the following proposition.

Proposition 6. *Assume $t^0 \notin \boldsymbol{\mathcal{S}}$ and consider the net transfer $r \in \boldsymbol{\mathcal{SM}}_R$.*

(i) Assume that $|\mathcal{C}^R| < \kappa$. Then there exists a blocking coalition $\mathcal{C} \subseteq \mathcal{C}^R$ for the transfer t^0 with corresponding $t \in \boldsymbol{\mathcal{S}}$ satisfying $t(S) = 0$ and $r^t = r$.

(ii) Assume that $|\mathcal{C}^R| \geq \kappa$ and $\Delta U_{\hat{\kappa}} \geq G^S > 0$, as described in Subsection 6.2.1. Then, there exists a blocking coalition $\mathcal{C} \subseteq \mathcal{C}^S$ for the transfer t^0 with corresponding $t \in \mathcal{A}^I$ and a blocking coalition $\tilde{\mathcal{C}} \subseteq \mathcal{C}^R$ for the transfer t with corresponding $\tilde{t} \in \mathcal{A}^I$, such that $r^{t+\tilde{t}} = r$ and $s^{t+\tilde{t}} = 0$.

The Proposition shows the construction of the sequence of blocking coalitions in some cases. Constructing the sequence in the case of higher order preemption described in Subsection 6.2.2 is more complex, and we do not develop it here. In the following example, we illustrate the construction in the case (ii) of Proposition 6.

Example 7. *Reconsider the committee with 4 members in Example 5, where decisions follow the majority rule $\kappa = 3$, and members have the utilities $u = (-5,\ 1,\ 2,\ 10)$. Without transfer, the reform is adopted $O(t^0) = R$. Consider the net transfer $r = (3,\ 1,\ 0,\ -4) \in \boldsymbol{\mathcal{SM}}_R$, which generates the payoff $\widehat{\pi}(t) = (-2,\ 2,\ 2,\ 6)$ for any t satisfying $r^t = r$. We proceed now to implement the net transfer r in two steps.*

In the first step, member 1 blocks the degenerate transfer t^0 with the deviation $t_{12}(S) = 1 + \varepsilon$ where ε is a small number. The resulting net transfers are $r^t = 0$ and $s^t = (-1 - \varepsilon,\ 1 + \varepsilon,\ 0,\ 0)$. Since $1 + \varepsilon > 1$, the committee decision is overturned,

$O(t) = S$, and the payoffs profile is $\widehat{\pi}(t) = (-1-\varepsilon,\ 1+\varepsilon,\ 0,\ 0)$. Since the utility of member 1 improves from -5 to $-1-\varepsilon$, member 1 forms the blocking coalition of the first step.

In the second step, member 2 and member 4 form a blocking coalition and promise the two-sided transfers $\tilde{t}$ defined by

$$\tilde{t}_{21}(S) = 1+\varepsilon,\ \tilde{t}_{41}(R) = 3,\ \tilde{t}_{42}(R) = 1,\ \text{and zero otherwise.}$$

Notice that $s^{t+\tilde{t}} = 0$ because member 2 deviates by returning the transfer promised by member 1 in the first step. This step is necessary since we aim to implement a one-sided transfer promise where all promises are contingent on reform adoption. The net transfer contingent on the reform is $r^{t+\tilde{t}} = (3,\ 1,\ 0,\ -4) \equiv r$. Hence $t+\tilde{t}$ implements the SM equilibrium r, and $\widehat{\pi}(t+\tilde{t}) = (-2,\ 2,\ 2,\ 6)$. As a last step, we need to verify that the coalition $\{2,4\}$ improves the payoff of its members after deviating. This holds true because member 2's payoff increases from $1+\epsilon$ to 2, while member 4's payoff rises from 0 to 6 and, hence the deviation $\tilde{t}$ results in a strict improvement of all coalition members' utility. As a result, the coalition $\{2,3\}$ blocks the transfer t with the deviation $\tilde{t}$. Thus we have implemented the equilibrium net transfer r. □

8 Conclusion

In this paper, we demonstrate the feasibility of analyzing pre-vote transfer promises contingent on vote outcome within committee settings governed by a quota rule. To obtain our results, we assume that the committee members can coordinate their action by forming coalitions, that all committee members can commit to make transfers, and that once the promises are done, it is impossible to renege on them. Under these assumptions, we show that strong Nash equilibria that minimize the total transfer exist. Thus, the practice of transfer promises contingent on vote outcome promotes efficiency and addresses the political failure of majority coercion. Importantly, we provide a detailed description of the type of transfers that are expected in equilibrium.

Understanding the types of transfers needed to achieve efficiency is crucial for designing institutions that facilitate the practice of promises and reduce the transaction costs associated to them. It is important to acknowledge that other forms of political failures, such as the tragedy of the commons (Olson Jr [1971]), voter ignorance (Downs [1957]), and rent-seeking (Tullock [1967]), are also significant and warrant further investigation. Therefore, there is still much work to be done in developing a comprehensive evaluation of the practice of transfer promises, particularly in environments where informational issues play a crucial role.

References

Alberto Alesina. Credibility and policy convergence in a two-party system with rational voters. *The American Economic Review*, 78(4):796–805, 1988.

Alberto Alesina and Stephen E. Spear. An overlapping generations model of electoral competition. *Journal of Public Economics*, 37(3):359–379, 1988.

Robert J Aumann. Acceptable points in general cooperative n-person games. *Contributions to the Theory of Games*, 4(40):287–324, 1959.

Marco Battaglini, Valerio Leone Sciabolazza, and Eleonora Patacchini. Logrolling in congress. Technical report, National Bureau of Economic Research, 2023.

Gary S Becker. Competition and democracy. *The Journal of Law and Economics*, 1: 105–109, 1958.

Duncan Black. *The Theory of Committees and Elections.*, volume 1. Cambridge University Press, 1958.

J.M. Buchanan and G. Tullock. *The Calculus of Consent, Logical Foundations of Constitutional Democracy*. University of Michigan Press, 1962.

Alessandra Casella. *Storable votes: protecting the minority voice*. Oxford University Press, 2012.

Alessandra Casella and Antonin Macé. Does vote trading improve welfare? *Annual Review of Economics*, 13:57–86, 2021.

Alessandra Casella and Thomas Palfrey. Trading votes for votes. a dynamic theory. *Econometrica*, 87(2):631–652, 2019.

Ying Chen and Jan Zápal. Sequential vote buying. *Journal of Economic Theory*, 205: 105529, 2022.

Ronald Coase. The problem of social cost. *The Journal of Law and Economics*, 3: 1–44, 1960.

Lauren Cohen and Christopher J Malloy. Friends in high places. *American Economic Journal: Economic Policy*, 6(3):63–91, 2014.

James S Coleman. The possibility of a social welfare function. *The American Economic Review*, 56(5):1105–1122, 1966.

Ernesto Dal Bo. Bribing voters. *American Journal of Political Science*, 51(4):789–803, 2007.

Eddie Dekel, Matthew O Jackson, and Asher Wolinsky. Vote buying: General elections. *Journal of Political Economy*, 116(2):351–380, 2008.

Eddie Dekel, Matthew O. Jackson, and Asher Wolinsky. Vote buying: legislatures and lobbying. *Quarterly Journal of Political Science*, 4(2):103–128, 2009.

Gerard Domènech-Gironell and Dimitrios Xefteris. Buying elections for peanuts. *Economic Theory*, pages 1–29, 2024.

Anthony Downs. An economic theory of political action in a democracy. *Journal of Political Economy*, 65(2):135–150, 1957.

Jon X. Eguia, Nicole Immorlica, Steven P. Lalley, Katrina Ligett, Glen Weyl, and Dimitrios Xefteris. Efficiency in collective decision-making via quadratic transfers, 2023. URL `https://arxiv.org/abs/2301.06206`.

Tore Ellingsen and Elena Paltseva. Confining the coase theorem: contracting, ownership, and free-riding. *The Review of Economic Studies*, 83(2):547–586, 2016.

Tim Groseclose and James M Snyder. Buying supermajorities. *American Political Science Review*, 90(2):303–315, 1996.

Henry TC Hu and Bernard Black. The new vote buying: Empty voting and hidden (morphable) ownership. *Southern California Law Review*, 79(4):811–908, 2005.

Matthew O Jackson and Simon Wilkie. Endogenous games and mechanisms: Side payments among players. *The Review of Economic Studies*, 72(2):543–566, 2005.

Charles Louis-Sidois and Leon Musolff. Buying voters with uncertain instrumental preferences. *Theoretical Economics*, 19(3):1305–1349, 2024.

Steven G Medema. The coase theorem at sixty. *Journal of Economic Literature*, 58 (4):1045–1128, 2020.

Roger B Myerson. Incentives to cultivate favored minorities under alternative electoral systems. *American Political Science Review*, 87(4):856–869, 1993.

Mancur Olson Jr. *The Logic of Collective Action: Public Goods and the Theory of Groups, with a new preface and appendix*, volume 124. Harvard University Press, 1971.

Tomas J Philipson and James M Snyder. Equilibrium and efficiency in an organized vote market. *Public Choice*, 89(3-4):245–265, 1996.

William H Riker and Steven J Brams. The paradox of vote trading. *American Political Science Review*, 67(4):1235–1247, 1973.

George J Stigler. *The Theory of Price.* Third edition, Macmillan, 1966.

Gordon Tullock. The welfare costs of tariffs, monopolies, and theft. *Economic Inquiry*, 5(3):224–232, 1967.

Alex Weingrod. Patrons, patronage, and political parties. *Comparative studies in Society and History*, 10(4):377–400, 1968.

Donald Wittman. Why democracies produce efficient results. *Journal of Political economy*, 97(6):1395–1424, 1989.

Dimitrios Xefteris and Nicholas Ziros. Strategic vote trading in power sharing systems. *American Economic Journal: Microeconomics*, 9(2):76–94, 2017.

Appendix: Proofs of the paper's propositions

Proof of Proposition 1: Suppose that $t \in \mathcal{A}^I$ is such that $m < \widehat{\kappa}$. Since $O(t) = R$, condition (i) in Proposition 1 implies that member σ_1 cannot gain by persuading the members $\sigma_{m+1}, \ldots, \sigma_{\widehat{\kappa}}$ to vote against the reform. Given that the coalition $\{\sigma_{m+1}, \ldots, \sigma_{\widehat{\kappa}}\}$ is the least costly to convince to vote against the reform, member σ_1 cannot induce any other coalition of size $\widehat{\kappa} - m$ to vote against it. Since member σ_1 has the strongest incentives to overturn the committee's decision, no other member can benefit from a Nash deviation. Conversely, if $t \in \boldsymbol{\mathcal{N}}$, then condition (i) is satisfied, indicating that member σ_1 cannot benefit from a Nash deviation. A similar argument holds for condition (ii) from Proposition 1.

Proof of Proposition 2: We proceed in four steps. We will use the notation $u_i^t = u_i + r_i^t - s_i^t$ for all $t \in \mathcal{A}$ and $i \in \mathbb{I}$.

Step 1. In this step we show that $\boldsymbol{\mathcal{S}}$ is nonempty. First, define $r \in \mathcal{P}$ by:

$$r_i = -u_i > 0, \quad i \leq n; \quad r_i := \frac{\sum_{j \leq n} u_j}{\sum_{j > n} u_j} u_i \leq 0, \ i > n.$$

A reasoning similar to that in Footnote 15 shows that there exists $t \in \mathcal{A}_R^I$ such that $r^t = r$. We now prove that $t \in \boldsymbol{\mathcal{S}}$. Note that $s^t \equiv 0$, and that $u_i^t := u_i + r_i$ for any i, we have

$$u_i^t = 0, \ i \leq n; \quad u_i^t = \left[1 + \frac{\sum_{j \leq n} u_j}{\sum_{j > n} u_j}\right] u_i = \frac{\sum_{j \in \mathbb{I}} u_j}{\sum_{j > n} u_j} u_i \geq 0, \ i > n.$$

Given the promise t, all members in $\mathbb{I}$ vote for R. The promise t is a strong equilibrium because any blocking coalition must defeat the reform, which would require the promisers' participation. However, since their utility was non-negative before the deviation and becomes non-positive after, they would not join. Thus, no blocking coalition exists, and $t \in \boldsymbol{\mathcal{S}}$.

Step 2. In this step, we show that $O(t) = R$ for all $t \in \boldsymbol{\mathcal{S}}$.

Assume by contradiction that $O(t) = S$ for some $t \in \boldsymbol{\mathcal{S}}$. Introduce

$$\tilde{r}_i := \overline{u}_* - u_i^t, \quad \text{where} \quad \overline{u}_* := \frac{1}{I}\sum_{j\in\mathbb{I}} u_j > 0.$$

Note that $\sum_{i\in\mathbb{I}} \tilde{r}_i = 0$, then there exists $\tilde{t} \in \mathcal{A}_R^I$ such that $r^{\tilde{t}} = \tilde{r}$. Note that

$$u_i^{t+\tilde{t}} = u_i^t + r_i^{\tilde{t}} - s_i^{\tilde{t}} = u_i^t + \tilde{r}_i = \overline{u}_* > 0, \quad i \in \mathbb{I}.$$

Then clearly $O(t+\tilde{t}) = R$. Moreover, since $O(t) = S$, we have

$$\hat{\pi}_i(t+\tilde{t}) - \hat{\pi}_i(t) = (u_i + r_i^{t+\tilde{t}}) - s_i^t = u_i^t + \tilde{r}_i = \overline{u}_* > 0, \quad i \in \mathbb{I}.$$

In particular, the above holds true for $i \in \mathcal{C} := \{i \in \mathbb{I} : \tilde{t}_i \neq 0\}$. That is, $\mathcal{C}$ is a blocking coalition for t, a desired contradiction with $t \in \boldsymbol{\mathcal{S}}$. Therefore, $O(t) = R$.

Step 3. We prove that if $t \in \boldsymbol{\mathcal{S}}$, then inequalities in the right side of (6) hold.

We proceed by contradiction and assume that there exists a coalition $\mathcal{C}$ satisfying $|\mathcal{C}| \geq \widehat{\kappa}$ such that $\sum_{i\in\mathcal{C}} u_i^t < 0$. Following similar arguments from Step 2, by restricting to all members in $\mathcal{C}$ and reversing the roles of R and S, there exist $\tilde{t} \in \mathcal{A}^I$ such that $\tilde{t}_{ij} = 0$ if either i or j is not in $\mathcal{C}$, $\tilde{t}(R) \equiv 0$, and $u_i^t + r_i^{\tilde{t}} = u_i^t < s_i^{\tilde{t}}$ for all $i \in \mathcal{C}$. Since $|\mathcal{C}| \geq \hat{\kappa}$, this implies that $O(t+\tilde{t}) = S$. From Step 1, we know $O(t) = R$, then

$$\hat{\pi}_i(t+\tilde{t}) - \hat{\pi}_i(t) = s_i^{t+\tilde{t}} - (u_i + r_i^t) = s_i^{\tilde{t}} - u_i^t > 0, \quad i \in \mathcal{C}.$$

In particular, the above holds true for $i \in \mathcal{C}' = \{i \in \mathcal{C} : \tilde{t}_i \neq 0\}$. Then we see that $\mathcal{C}'$ is a blocking coalition for t, contradicting with the assumption that $t \in \boldsymbol{\mathcal{S}}$.

Step 4. We prove that if a promises profile $t \in \mathcal{A}^I$ satisfies the inequalities (6) for every coalition of size $|\mathcal{C}| \geq \widehat{\kappa}$, then $t \in \boldsymbol{\mathcal{S}}$.

First, observe that if inequalities (6) hold, then $O(t) = R$. This is because if $O(t) = S$, then there must exist a coalition $|\mathcal{C}| \geq \widehat{\kappa}$ whose members vote unanimously for the status quo. This cannot be true because when $\sum_{\mathcal{C}}(u_i + r_i^t) \geq \sum_{\mathcal{C}} s_i^t$, we cannot have $u_i + r_i^t < s_i^t$ for all $i \in \mathcal{C}$.

To prove that $t \in \boldsymbol{\mathcal{S}}$, we proceed again by contradiction and assume the opposite, that is, there exists a blocking coalition $\mathcal{C}$ for t, with corresponding incremental transfer $\tilde{t} \in \mathcal{A}^I$. Since $O(t) = R$, any blocking coalition $\mathcal{C}$ must overturn the reform: $O(t+\tilde{t}) = S$. That is, $|\tilde{\mathcal{C}}| \geq \hat{\kappa}$, where $\tilde{\mathcal{C}} := \{i : u_i^{t+\tilde{t}} < 0\}$. Notice that for all $i \in \tilde{\mathcal{C}}$, we have $u_i^t < s_i^{\tilde{t}} - r_i^{\tilde{t}}$.

Moreover, by the definition of blocking coalition, we have $\hat{\pi}_i(t+\tilde{t}) = s_i^{t+\tilde{t}} > u_i + r_i^t = \hat{\pi}_i(t)$, for all $i \in \mathcal{C}$. Hence, $u_i^t < s_i^{\tilde{t}}$ for all $i \in \mathcal{C}$.

Noticing that $|\mathcal{C} \cup \tilde{\mathcal{C}}| \geq |\tilde{\mathcal{C}}| \geq \hat{\kappa}$, the inequality (6) applied to $\mathcal{C} \cup \tilde{\mathcal{C}}$ gives

$$\begin{aligned} 0 &\leq \sum_{i \in \mathcal{C} \cup \tilde{\mathcal{C}}} u_i^t = \sum_{i \in \mathcal{C}} u_i^t + \sum_{i \in \tilde{\mathcal{C}} \setminus \mathcal{C}} u_i^t \\ &< \sum_{i \in \mathcal{C}} s_i^{\tilde{t}} + \sum_{i \in \tilde{\mathcal{C}} \setminus \mathcal{C}} (s_i^{\tilde{t}} - r_i^{\tilde{t}}) = \sum_{i \in \mathcal{C} \cup \tilde{\mathcal{C}}} s_i^{\tilde{t}} - \sum_{i \in \tilde{\mathcal{C}} \setminus \mathcal{C}} r_i^{\tilde{t}} = - \sum_{i \notin \mathcal{C} \cup \tilde{\mathcal{C}}} s_i^{\tilde{t}} - \sum_{i \in \tilde{\mathcal{C}} \setminus \mathcal{C}} r_i^{\tilde{t}}, \end{aligned} \tag{15}$$

where the last inequality is due to the fact that $s^{\tilde{t}} \in \mathcal{P}$. But the last term of this inequality must be non-positive because $r_i^{\tilde{t}} \geq 0$ and $s_i^{\tilde{t}} \geq 0$ for all i that are outside the blocking coalition $\mathcal{C}$. This is because only members of the blocking coalition can initiate promises, while outsiders are either recipients of promises or do not receive any transfers. Therefore we get a contradiction since the last term of (15) must be positive and at the same time non-negative. □

Proof of Proposition 3: First, note that $\mathcal{S}_R = \{r^t - s^t : t \in \boldsymbol{\mathcal{S}}\}$. Then by Proposition 2 we see that $\boldsymbol{\mathcal{S}}_R$ is non-empty and closed, with

$$\boldsymbol{\mathcal{S}}_R = \Big\{ r \in \mathcal{P} : \sum_{i \in \mathcal{C}} (u_i + r_i) \geq 0 \text{ for all coalitions } |\mathcal{C}| \geq \hat{\kappa} \Big\}. \tag{16}$$

Fix an arbitrary $r^0 \in \boldsymbol{\mathcal{S}}_R$, and denote $\boldsymbol{\mathcal{S}}_R(r^0) := \{r \in \boldsymbol{\mathcal{S}}_R : \mathcal{T}_r \leq \mathcal{T}_{r^0}\}$. Since $\mathcal{T}_r > \mathcal{T}_{r^0}$ for $r \in \boldsymbol{\mathcal{S}}_R \backslash \boldsymbol{\mathcal{S}}_R(r^0)$, then $\boldsymbol{\mathcal{SM}}_R = \{r \in \boldsymbol{\mathcal{S}}_R(r_0) : \mathcal{T}_r = \inf_{r' \in \boldsymbol{\mathcal{S}}_R(r^0)} \mathcal{T}_{r'}\}$. The continuity of the function $r \to \mathcal{T}_r$ implies that $\boldsymbol{\mathcal{S}}_R(r^0)$ is closed and bounded, and thus compact. Consequently, the set of minimizers $\boldsymbol{\mathcal{SM}}_R$ is non-empty and compact. Finally, for any $r^1, r^2 \in \boldsymbol{\mathcal{SM}}_R \subset \boldsymbol{\mathcal{S}}_R$ and any $0 < \alpha < 1$, denote the convex combination $r := \alpha r^1 + (1-\alpha) r^2 \in \mathcal{P}$. Note that if r^1, r^2 satisfy (16), the net transfer r also satisfies (16) and hence $r \in \boldsymbol{\mathcal{S}}_R$. Moreover, since r^1, r^2 are minimizers, we have $\mathcal{T}_r \leq \alpha \mathcal{T}_{r^1} + (1-\alpha)\mathcal{T}_{r^2} = \inf_{r' \in \boldsymbol{\mathcal{S}}_R} \mathcal{T}_{r'}$. Then r is also a minimizer, and thus $r \in \boldsymbol{\mathcal{SM}}_R$. This proves that $\boldsymbol{\mathcal{SM}}_R$ is convex. □

Proof of Proposition 4: In the subsequent proofs, we will work directly with the net transfers $r \in \mathcal{P}$ since the characterization of strong Nash equilibria from Proposition 2 only depend on the net transfers $r^t - s^t$. We will use the abuse of notation $O(r) = O(t)$, $\widehat{\pi}(r) = \widehat{\pi}(t)$ with the understanding that $r^t = r$.

The proof relies on the following three lemmas.

Lemma 1. *For any equilibrium net transfer promises $r \in \boldsymbol{\mathcal{SM}}_R \backslash \{0\}$, there exists no pair of members (i, j) such that*

$$r_i < 0 < r_j \quad \textit{and} \quad \widehat{\pi}_i(r) < \widehat{\pi}_j(r). \tag{17}$$

Proof. Assume by contradiction that (17) holds true. For some small $\varepsilon > 0$, set

$$\tilde{r}_i = r_i + \varepsilon \leq 0, \quad \tilde{r}_j = r_j - \varepsilon \geq 0, \quad \text{and} \quad \tilde{r}_k = r_k \text{ for all } k \neq i, j. \tag{18}$$

Notice $\sum_{\mathbb{I}} \tilde{r}_i = \sum_{\mathbb{I}} r_i = 0$ and hence, $\tilde{r} \in \mathcal{P}$. Next, for any $|\mathcal{C}| \geq \widehat{\kappa}$, since $r \in \boldsymbol{\mathcal{SM}}_R \subset \boldsymbol{\mathcal{S}}_R$, using (16) and inequality (17) gives

$$\sum_{k \in \mathcal{C}} (u_k + \tilde{r}_k) = \begin{cases} \displaystyle\sum_{k \in \mathcal{C}} (u_k + r_k) \geq 0, & \text{if } i, j \in \mathcal{C} \text{ or } i, j \notin \mathcal{C}; \\ \displaystyle\sum_{k \in \mathcal{C}} (u_k + r_k) + \varepsilon > \sum_{k \in \mathcal{C}} (u_k + r_k) \geq 0, & \text{if } i \in \mathcal{C}, j \notin \mathcal{C}. \end{cases} \tag{19}$$

For the last case that $j \in \mathcal{C}, i \notin \mathcal{C}$, by setting $\varepsilon < \widehat{\pi}_j(r) - \widehat{\pi}_i(r)$, we have $\widehat{\pi}_i(r) < \widehat{\pi}_j(r) - \varepsilon = u_j + \tilde{r}_j$. Then, since the cardinal of the coalition $(\mathcal{C}\backslash\{j\}) \cup \{i\}$ is also larger than $\widehat{\kappa}$, we have

$$\sum_{k\in\mathcal{C}}(u_k+\tilde{r}_k) \geq \sum_{k\in(\mathcal{C}\setminus\{j\})\cup\{i\}}(u_k+\tilde{r}_k) \geq 0, \quad \text{if } j\in\mathcal{C}, i\notin\mathcal{C}. \tag{20}$$

Combining (19) and (20) and recalling that $\mathcal{C}$ is an arbitrary coalition satisfying $|\mathcal{C}|\geq\widehat{\kappa}$, by (16) again we see that $\tilde{r}\in\boldsymbol{\mathcal{S}}_R$. Note further that

$$|\tilde{r}_i| = -\tilde{r}_i = -r_i-\varepsilon = |r_i|-\varepsilon, \quad |\tilde{r}_j| = \tilde{r}_j = r_j-\varepsilon = |r_j|-\varepsilon, \quad |\tilde{r}_k| = |r_k|,\ k\neq i,j.$$

Then

$$\mathcal{T}_{\tilde{r}} = \frac{1}{2}\sum_{k=1}^{I}|\tilde{r}_k| = \frac{1}{2}\Big[\sum_{k\neq i,j}|r_k|+|r_i|-\varepsilon+|r_j|-\varepsilon\Big] = \mathcal{T}_r-\varepsilon<\mathcal{T}_r.$$

This contradicts the minimum total promises transfer property of $r\in\boldsymbol{\mathcal{SM}}_R$. □

Lemma 2. *For any equilibrium promises profile $r\in\boldsymbol{\mathcal{SM}}_R\setminus\{0\}$, there exists no committee member i such that*

$$r_i<0 \quad \textit{and} \quad \widehat{\pi}_i(r)<0. \tag{21}$$

Proof. Assume by contradiction that (21) holds true. Since $r\in\mathcal{P}$, there exists $j\neq i$ such that $r_j>0$. Then by Lemma 1, we have $\widehat{\pi}_j(r)\leq\widehat{\pi}_i(r)<0$. Define the net transfer $\tilde{r}$ by (18) again. Note that $u_i+\tilde{r}_i=\widehat{\pi}_i(r)+\varepsilon$, $u_j+\tilde{r}_j=\widehat{\pi}_j(r)-\varepsilon$. Then for $\varepsilon>0$ small enough, we have $u_j+\tilde{r}_j<u_i+\tilde{r}_i\leq 0$.

Following the same reasoning as in Lemma 1, we prove now that the inequality $\sum_{k\in\mathcal{C}}(u_k+\tilde{r}_k)\geq 0$ must hold for every coalition $|\mathcal{C}|\geq\widehat{\kappa}$. First, observe that by (16), we have $\sum_{k\in\mathcal{C}}\widehat{\pi}_k(r)\geq 0$ and hence, there exists $m\in\mathcal{C}$ such that $\widehat{\pi}_m(r)\geq 0$. Notice that, since $\widehat{\pi}_j(r)\leq\widehat{\pi}_i(r)<0$, we have $m\neq i,j$.

When $i,j\in\mathcal{C}$, or $i,j\notin\mathcal{C}$, or $i\in\mathcal{C}, j\notin\mathcal{C}$, (19) remains true and hence $\sum_{k\in\mathcal{C}}(u_k+\tilde{r}_k)\geq 0$. In the last case where $j\in\mathcal{C}, i\notin\mathcal{C}$, since $u_m+\tilde{r}_m=u_m+r_m\geq 0\geq u_i+\tilde{r}_i$ and $|\mathcal{C}\setminus\{m\})\cup\{i\}|\geq\widehat{\kappa}$, we have

$$\sum_{k\in\mathcal{C}}(u_k+\tilde{r}_k) \geq \sum_{k\in(\mathcal{C}\setminus\{m\})\cup\{i\}}(u_k+\tilde{r}_k) = \sum_{k\in(\mathcal{C}\setminus\{m\})\cup\{i\}}(u_k+r_k)\geq 0.$$

This, together with equations (19) and (16), implies $\tilde{r} \in \boldsymbol{\mathcal{S}}_R$. Then, following the same argument on the total transfer used in the last step of the proof of Lemma 1, we derive the desired contradiction. □

Lemma 3. *For any equilibrium promises profile $r \in \boldsymbol{\mathcal{S}} \backslash \{0\}$, introduce*

$$\underline{k}_* := \max\{i | r_i > 0\}, \qquad \overline{k}_* := \min\{i | r_i < 0\}. \tag{22}$$

Then

$$\underline{k}_* < \overline{k}_*, \quad \text{and} \quad r_k = 0 \quad \text{for all } \underline{k}^* < k < \overline{k}_*. \tag{23}$$

Committee member $\overline{k}_*$ is a reform supporter, $\overline{k}_* \in \mathcal{C}^R$, none of the committee members $j \geq \overline{k}_*$ is transfer recipient and when they are promisers, they do not transfer more than their utility,

$$-u_j \leq r_j \leq 0, \text{ for all } j \geq \overline{k}_*. \tag{24}$$

None of the committee members $i \leq \underline{k}_*$ is a net promiser

$$r_i \geq 0, \text{ for all } i \leq \underline{k}_*. \tag{25}$$

The post-transfer utilities $\widehat{\pi}(r) \equiv u + r$ are ranked across the coalition of promisers and the coalition of promisees, that is,

$$\widehat{\pi}_i(r) \leq \widehat{\pi}_j(r) \quad \text{for all} \quad i \leq \underline{k}_* < \overline{k}_* \leq j. \tag{26}$$

Proof. First, since $r_{\overline{k}_*} < 0$, by Lemma 2 we have $u_{\overline{k}_*} + r_{\overline{k}_*} \geq 0$, which implies $u_{\overline{k}_*} > 0$. That is, $\overline{k}_* \in \mathcal{C}^R$.

Next, since $r_{\overline{k}_*} < 0 < r_{\underline{k}_*}$, by Lemma 1 we have $u_{\underline{k}_*} + r_{\underline{k}_*} \leq u_{\overline{k}_*} + r_{\overline{k}_*}$, and thus $u_{\underline{k}_*} < u_{\overline{k}_*}$. Then by the ranking condition $u_1 \leq u_2 \leq \cdots \leq u_I$, we obtain $\underline{k}_* < \overline{k}_*$. The inequality $\underline{k}_* < \overline{k}_*$ and the definitions (22) imply that $r_k = 0$ for all $\underline{k}^* < k < \overline{k}_*$, and this proves the statement in (23).

We now prove the statements in (25). Assume by contradiction that $r_i < 0$ for some $i \leq \underline{k}_*$. Since $r_{\underline{k}_*} > 0$, we must have $i < \underline{k}_*$. Since the vector u is ranked, we have

$\widehat{\pi}_i(r) < u_i \leq u_{\underline{k}_*} < \widehat{\pi}_{\underline{k}_*}(r)$. This contradicts Lemma 1, and thus $r_i \geq 0$ for all $i \leq \underline{k}_*$. Similarly, to prove the inequalities (24), assume there exists $j \geq \overline{k}_*$ such that $r_j > 0$. Then we would have $r_{\overline{k}_*} < 0 < r_j$ and $\widehat{\pi}_{\overline{k}_*}(r) < u_{\overline{k}_*} \leq u_j < \widehat{\pi}_j(r)$. This contradicts Lemma 1, and thus $r_j \leq 0$ for all $j \geq \overline{k}_*$. Moreover, if $r_j < -u_j$ for some $j \geq \overline{k}_*$, then $\widehat{\pi}_j(r) = u_j + r_j < 0$. Since $\overline{k}_* \in \mathcal{C}^R$, then $j \in \mathcal{C}^R$, and thus $r_j < -u_j \leq 0$. To sum up, $r_j < 0$ and $\widehat{\pi}_j(r) < 0$. This contradicts Lemma 2, so $r_j \geq -u_j$ for all $j \geq \overline{k}_*$, and thus (24) holds true.

We finally prove inequalities (26). Assume by contradiction that $\widehat{\pi}_j(r) < \widehat{\pi}_i(r)$ for some $i \leq \underline{k}_* < \overline{k}_* \leq j$. By (25) we have $r_j \leq 0 \leq r_i$. If $r_j < 0 < r_i$, we obtain the contradiction with Lemma 1. If $r_j = 0 < r_i$, we have $r_{\overline{k}_*} < 0 < r_i$, and since the utility vector is ordered, we have $\widehat{\pi}_{\overline{k}_*}(r) < u_{\overline{k}_*} \leq u_j = \widehat{\pi}_j(r) < \widehat{\pi}_i(r)$, contradicting Lemma 1. Similarly, if $r_j < 0 = r_i$, we have $r_j < 0 < r_{\underline{k}_*}$, and, since the utility vector is ordered, we have $\widehat{\pi}_{\underline{k}_*}(r) > u_{\underline{k}_*} \geq u_i = \widehat{\pi}_i(r) > \widehat{\pi}_j(r)$, also contradicting Lemma 1. In the last case that $r_i = 0 = r_j$, we have $r_{\overline{k}_*} < 0 < r_{\underline{k}_*}$, and hence $\widehat{\pi}_{\overline{k}_*}(r) < u_{\overline{k}_*} \leq u_j = \widehat{\pi}_j(r) < \widehat{\pi}_i(r) = u_i \leq u_{\underline{k}_*} < \widehat{\pi}_{\underline{k}_*}(r)$. This again contradicts Lemma 1. In summary, we obtain a contradiction with Lemma 1 in all the sub-cases, and thus (26) holds true. $\square$

Proof of Proposition 4: We note that, from the proof below, we see that in all the cases $\underline{k}_* < k_* \leq \overline{k}_*$. In this proof, we allow k_* to depend on $r \in \boldsymbol{\mathcal{SM}}_R \backslash \{0\}$. In the supplemental Appendix A, we show that a common k_* can be selected for all $r \in \boldsymbol{\mathcal{SM}}_R \backslash \{0\}$.

We first prove the statements in (8). Define $a := \min_{\overline{k}_* \leq j \leq I} \widehat{\pi}_j(r)$. By (23) we have $\widehat{\pi}_k(r) = u_k$ for $\underline{k}_* < k < \overline{k}_*$. If $a \geq \widehat{\pi}_{\overline{k}_*-1}(r)$, then set $k_* = \overline{k}_*$. Since the utility vector is ordered, we have $a \geq \widehat{\pi}_{\overline{k}_*-1}(r) \geq \widehat{\pi}_k(r)$ for all $\underline{k}_* < k < \overline{k}_*$, and by (26) we have $a \geq \widehat{\pi}_i(r)$ for all $i \leq \underline{k}_*$. Moreover, by (23) and (26) we see that $r_i \geq 0 \geq r_j$ for all $i < k_* \leq j$. So $k_* = \overline{k}_*$ satisfies all the requirements in inequalities (8). We next assume $a < \widehat{\pi}_{\overline{k}_*-1}(r)$. Since $a \geq \widehat{\pi}_{\underline{k}_*}(r)$, we may set $k_* = \inf\{k > \underline{k}_* : \widehat{\pi}_k(r) > a\}$. Then $\underline{k}_* < k_* \leq \overline{k}_*$, and since the utility vector is ordered, we see that $\widehat{\pi}_k(r) \leq a$

for all $\underline{k}_* < k < k_*$, and $\widehat{\pi}_k(r) \geq \widehat{\pi}_{k_*}(r) > a$ for all $k_* \leq k < \overline{k}_*$. Recall again (26), then $\widehat{\pi}_i(r) \leq a \leq \widehat{\pi}_j(r)$ for all $i < k_* \leq j$, and by (23) and (25), we have $r_i \geq 0 \geq r_j$ for all $i < k_* \leq j$. This completes the proof of (8). Moreover, since $r \in \mathcal{P}$, then $\sum_{i<k_*} r_i = \sum_{j \geq k_*}(-r_j)$, and thus we obtain (9). □

Proof of Proposition 5: Assume that there exists a strong Nash equilibrium with retraction $t \in \mathcal{A}^I$. A strong equilibrium with retraction is necessarily a strong Nash equilibrium. Proposition 2 implies that $O(t) = R$ and, $\sum_{\mathcal{C}}(u_i + r_i^t - s_i^t) \geq 0$ for all coalition $\mathcal{C}$ satisfying $|\mathcal{C}| \geq 2$.

Assume first that t is a one-sided transfer contingent on the status quo, $t(R) = 0$. Define the deviation $\tilde{t}_{1j}(S) = \tilde{t}_{2j}(S) = 0$, for all j and, $\tilde{t} = t$ for all other combinations of (ij) and $O \in \{R, S\}$. Since $O(t) = R$, we have $\widehat{\pi}_1(t) = -2$ and $\widehat{\pi}_2(t) = -1$. On the other hand, for $i = 1, 2$, we have $s_i^{\tilde{t}} = \tilde{t}_{3i}(S) \equiv t_{3i}(S) \geq 0$. Thus members 1 and 2 vote for the status quo and $O(\tilde{t}) = S$. Then, for $i = 1, 2$, we have $\widehat{\pi}_i(\tilde{t}) = s_i^{\tilde{t}} \geq 0 > \widehat{\pi}_i(t)$. Therefore the coalition $\{1, 2\}$ blocks the transfer function t with the retracting deviation $\tilde{t}$. This is a contradiction.

We next assume $t(R) \neq 0$; that is, there exist $i, j = 1, 2, 3$ such that $i \neq j$ and $t_{ij}(R) > 0$. We will then consider two cases.

Case 1: $t_{ij}(R) > 0$ and $u_j + r_j^t > s_j^t$. Let us fix ε such that $\varepsilon < t_{ij}(R)$ and, $\varepsilon < u_j + r_j^t - s_j^t$. Consider the retracting deviation $\tilde{t}$ where member i decreases her transfers to member j by ε. We have $\tilde{t}_{ij}(R) = t_{ij}(R) - \varepsilon$ and the transfer t and $\tilde{t}$ are equal in all other cases. If we denote $\{k\}$, the complement of $\{i, j\}$ in the set $\{1, 2, 3\}$, we have, $r_j^{\tilde{t}} = r_j^t - \varepsilon$, $r_i^{\tilde{t}} = r_i^t + \varepsilon$, $r_k^{\tilde{t}} = r_k^t$ and, $s^{\tilde{t}} = s^t$. We have

$$u_i + r_i^{\tilde{t}} = u_i + r_i^t + \varepsilon > u_i + r_i^t = \widehat{\pi}_i(t); \tag{27}$$

$$u_j + r_j^{\tilde{t}} = u_j + r_j^t - \varepsilon > s_j^t = s_j^{\tilde{t}}. \tag{28}$$

Equation (28) shows that member j votes for R after the deviation $\tilde{t}$, even though her incentives to support R have diminished by ε compared to before the deviation. On the other hand, at least one member from the pair (i, k) votes for R after the deviations. This is because the incentive to vote for R is raised by ε for member i and is unchanged for member k. Since $O(t) = R$, we know that at least one of them votes for R prior to the deviation and, therefore, that member will still vote for R after the deviation. To sum up, member j and at least one member of the pair $\{i, k\}$ vote for R after the deviation and hence, $O(\tilde{t}) = R$. Equation (27) implies that $\widehat{\pi}_i(\tilde{t}) = u_i + r_i^{\tilde{t}} > \widehat{\pi}_i(t)$. Therefore the singleton coalition $\{i\}$ blocks the transfer t with the deviation $\tilde{t}$, contradicting that t is a strong equilibrium with retraction.

Case 2: $t_{ij}(R) > 0$ and $u_i + r_i^t \leq s_i^t$. The transfer t is a strong equilibrium with retraction, we must have $t \in \boldsymbol{\mathcal{S}}$ and the inequalities (6) hold for any coalition that includes i and a second member. Therefore, we have $u_{j'} + r_{j'}^t \geq s_{j'}^t$ for all $j' \neq i$. Consider the retracting deviation $\tilde{t}$ where member i decreases her transfers to member j by ε, where ε is defined in the previous step of this proof. Then $u_j + r_j^{\tilde{t}} = u_j + r_j^t - \varepsilon > s_j^t = s_j^{\tilde{t}}$ and therefore member j votes for S after the deviation. Denoting again by $\{k\} := \{1, 2, 3\} \backslash \{i, j\}$, and recalling that $u_{j'} + r_{j'}^t \geq s_{j'}^t$ for all $j' \neq j$, we have

$$u_i + r_i^{\tilde{t}} = u_i + r_i^t + \varepsilon > s_i^t = s_i^{\tilde{t}}, \qquad u_k + r_k^{\tilde{t}} = u_k + r_k^t > s_k^t = s_k^{\tilde{t}}.$$

Thus both members i and k vote for R and hence $O(\tilde{t}) = R$. We have $\widehat{\pi}_i(\tilde{t}) = u_i + r_i^{\tilde{t}} = u_i + r_i^t + \varepsilon > u_i + r_i^t = \widehat{\pi}_i(t)$. Again, the singleton coalition $\{i\}$ blocks the transfer t with the deviation $\tilde{t}$, contradicting that t is a strong equilibrium with retraction. $\square$

Proof of Proposition 6: In order to avoid technicalities, we make the following two relaxations in our convention and definitions. Firstly, we assume that, when a member i is indifferent between R and S, we assume he would vote for S in the first step of our sequence of blocking coalitions. Second, we remove the strict dominance for a blocking coalition $\mathcal{C}$ and only require a Pareto improvement, that is, $\hat{\pi}_i(t + \tilde{t}) \geq \hat{\pi}_i(t)$ for a given deviation $\tilde{t}$ and for all $i \in \mathcal{C}$, and with strict inequality for some $i \in \mathcal{C}$.[21]

[21] To understand why these relaxations are needed in some cases, assume $I = 3$, $\kappa = 3$, $u = (-2, 1, 2)$. Then the net transfer $r = (2, -1, -1) \in \boldsymbol{\mathcal{SM}}_R$ with $\hat{\pi}(r) = (0, 0, 1)$. However, in this case, member 2 does not strictly increase his payoff and thus cannot be a member of a blocking coalition if we require

(i) By setting $t(S) \equiv 0$ and $r^t = r$, we have $u + r^t = u + r$. Since $r \in \boldsymbol{\mathcal{SM}}_R$, we have $|\mathcal{C}| \geq \kappa$, where $\mathcal{C} := \{i : u_i + r_i \geq 0\}$. Then $O(t) = R$, and $\hat{\pi}_i(t) = u_i + r_i \geq 0 = s_i^{t^0} = \hat{\pi}_i(t^0)$ for all $i \in \mathcal{C}$. Moreover, since $\sum_{i\in\mathbb{I}}(u_i + r_i) = \sum_{i\in\mathbb{I}} u_i > 0$, there exists $i \in \mathcal{C}$ such that $u_i + r_i > 0$, which implies $\hat{\pi}_i(t) > \hat{\pi}_i(t^0)$. This implies $\mathcal{C}$ is a blocking coalition for t^0 with corresponding t, in our relaxed sense.

(ii) **Step 1.** First, denote $\alpha := \frac{\sum_{j\in\underline{\mathcal{C}}^R} u_j}{\sum_{j\in\mathcal{C}^S} |u_j|} > 0$ and define $s \in \mathcal{P}$ by

$$s_i = \alpha u_i < 0,\ i \in \mathcal{C}^S; \quad s_i := u_i \geq 0,\ i \in \underline{\mathcal{C}}^R; \quad s_i := 0,\ i \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R,$$

There exists $t \in \mathcal{A}^I$ such that $t(R) \equiv 0$, $s^t = s$ and $t_{ij}(S) > 0$ can hold only when $i \in \mathcal{C}^S$ and $j \in \underline{\mathcal{C}}^R$, that is, the promises are made only from members in $\mathcal{C}^S$ to members in $\underline{\mathcal{C}}^R$. We now verify that $\mathcal{C}^S$ is a relaxed blocking coalition for t^0 with the promise profile t. First, since $G^S > 0$, we have $0 \leq \sum_{j\in\underline{\mathcal{C}}^R} u_j < \sum_{j\in\mathcal{C}^S} |u_i|$. Then $\alpha < 1$, and thus $u_i < s_i$ for $i \in \mathcal{C}^S$. This implies that

$$u_i^t = u_i - s_i < 0,\ i \in \mathcal{C}^S; \quad u_i^t = u_i - s_i = 0,\ i \in \underline{\mathcal{C}}^R.$$

Due to our relaxation, $|\mathcal{C}^S \cup \underline{\mathcal{C}}^R| = \hat{\kappa}$ implies $O(t) = S$ Moreover, $O(t^0) = R$ implies $\hat{\pi}_i(t) - \hat{\pi}_i(t^0) = s_i - u_i = (\alpha - 1)u_i > 0$ for any $i \in \mathcal{C}^S$. Thus the coalition $\mathcal{C}^S$ blocks t^0.

Step 2. In this step we show that $\tilde{\mathcal{C}} := \mathcal{C}^R$ is a blocking coalition for t. We first let the members in $\underline{\mathcal{C}}^R$ to return s^t to the members in $\mathcal{C}^S$: $\tilde{t}(S) := -t(S)$. Next, noting that $r^t = 0$, we set $\tilde{t}(R)$ be such that $r^{\tilde{t}} = r$. Then $r^{t+\tilde{t}} - s^{t+\tilde{t}} = r$. Since $r \in \boldsymbol{\mathcal{SM}}_R$, then $O(t + \tilde{t}) = R$. Moreover, recalling that $O(t) = S$, we have for all $i \in \mathcal{C}^R$,

$$\hat{\pi}_i(t + \tilde{t}) - \hat{\pi}_i(t) = u_i + r_i^{t+\tilde{t}} - s_i^t = u_i + r_i - s_i = \begin{cases} u_i + r_i - u_i = r_i \geq 0,\ i \in \underline{\mathcal{C}}^R; \\ u_i + r_i \geq 0,\ i \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R. \end{cases}$$

Since $\sum_{i\in\mathbb{I}}(u_i + r_i) = \sum_{i\in\mathbb{I}} u_i > 0$, by the order preserving property (8) we must have $u_i + r_i > 0$ for some $i \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R$. This shows that $t + \tilde{t}$ Pareto dominates t on $\tilde{\mathcal{C}}$. Then, in our relaxed sense, $\tilde{\mathcal{C}}$ is a blocking coalition for t with corresponding $\tilde{t}$. ☐

strict utility improvements for all members of blocking coalitions. One way to avoid this issue is to replace the strict dominance with Pareto dominance. Alternatively, if we insist on requiring a strict dominance of blocking coalitions, we need to define ε-equilibria, and the proof will become technical.

Supplemental Appendix for:

"Democratic Policy Decisions with Decentralized Promises Contingent on Vote Outcome"

by Ali Lazrak and Jianfeng Zhang

Supplemental Appendix A (SA): SM equilibria implications

In this Appendix, we characterize SM equilibria for all subcases discussed in Section 6 of the paper using numerical examples. In all subsequent discussions, we will work directly with the net transfers $r \in \mathcal{P}$ since the characterization of strong Nash equilibria from Proposition 2 only depends on the net transfers $r^t - s^t$. We will use the abuse of notation $O(r) = O(t)$, $\widehat{\pi}(r) = \widehat{\pi}(t)$ with the understanding that $r^t = r$ and $t(S) \equiv 0$.

SA.1 The case of "majority coercion": $\mid \mathcal{C}^R \mid < \kappa$

Proposition SA.1. *[**Equilibrium transfer promises with majority coercion**] Consider a committee with weak support for the reform, $\mid \mathcal{C}^R \mid < \kappa$ with $\kappa \geq 2$ resulting in the reform's defeat, $O(t^0) = S$.*[22] *A net transfer profile $r \in \mathcal{P}$ is an SM equilibrium, $r \in \boldsymbol{\mathcal{SM}}_R$, if and only if*

1. *The promises are across the aisle: the promise recipients are reform opponents and the promisers are reform supporters. Moreover the total transfer is given by $\mathcal{T}_r = U^S = \sum_{\mathcal{C}^S} r_i = -\sum_{\mathcal{C}^R} r_i$.*

2. *For all $i \in \mathcal{C}^R$, the individual rationality constraint $-u_i \leq r_i \leq 0$ holds.*

3. *All promise recipients are reform opponents, and the net transfers $(r_i)_{i \in \mathcal{C}^S}$ satisfy:*

   (a) *When $|\mathcal{C}^R| < \kappa - 1$, each member $i \in \mathcal{C}^S$ is promised the transfer $r_i = -u_i > 0$ just to make her indifferent between the reform and the status quo, $\widehat{\pi}_i(r) = 0$.*

   (b) *When $|\mathcal{C}^R| = \kappa - 1$, the net transfer of the members of $\mathcal{C}^S$ are non-negative, $r_i \geq 0$ for all $i \in \mathcal{C}^S$ and the post-transfer utilities of members in $\mathcal{C}^S$ cannot exceed those of members in $\mathcal{C}^R$: $\widehat{\pi}_i(r) \leq \widehat{\pi}_j(r)$ for any $i \in \mathcal{C}^S$ and $j \in \mathcal{C}^R$.*

[22] Assumption (2) implies that at least one member supports the reform. The case $\kappa = 1$ can be excluded from the proposition, as the reform always passes in this scenario, resulting in no voting inefficiencies.

In all SM equilibria, the total transfer compensates reform opponents for the aggregate disutility that they experience with the passage of the reform. The critical voter k_* from Proposition 4 is given by $k_* = n + 1$, so that the reform supporters are promisers and reform opponents are promisees. The equilibrium leaves multiple ways for the members of the coalition $\mathcal{C}^R$ to divide among themselves the total transfer directed towards reform opponents. Whether there is an additional indeterminacy on the side of the promises recipients depends on the size of the coalition $\mathcal{C}^R$.

If the coalition $\mathcal{C}^R$ is short at least 2 members of being decisive ($|\mathcal{C}^R| < \kappa - 1$), then the distribution of equilibrium promises among promisees is unique. After receiving the promises, all members of $\mathcal{C}^S$ are indifferent between voting for or against the reform.

When the coalition $\mathcal{C}^R$ is just one member short of being decisive ($|\mathcal{C}^R| = \kappa - 1$), the transfers to the members of $\mathcal{C}^S$ are indeterminate. In that case, multiple distributions of promises across the members of the receiving coalition $\mathcal{C}^S$ can form an equilibrium provided that they satisfy the requirement 3.(b) of Proposition SA.1. The requirement restricts the promises directed to each member of the coalition $\mathcal{C}^S$ to produce post-transfer utilities that maintain the ordering across the coalitions of promisers and promisees of the utilities prior to the transfer. Changing the ordering of utilities across the coalition $\mathcal{C}^R$ and $\mathcal{C}^S$ would create the incentives to engage in additional rounds of promises and contradict the stability requirement of the equilibrium. When $|\mathcal{C}^R| = \kappa - 1$, the committee adopts the reform after the promises are made, but the vote for the reform may not be unanimous.

SA.2 Committee with strong support for the reform: $\mid \mathcal{C}^R \mid \geq \kappa$

In this subsection, we study the equilibrium when the committee would adopt the reform in the absence of promises. First, the following results identify the unique equilibrium promises profile when there are no gains from trade.

Proposition SA.2. *[**No promises equilibrium in the absence of gains from trade**] Consider a committee with more reform supporters than the κ-majority requirement, $|\, \mathcal{C}^R \,| \geq \kappa$. Assume the gain from trade defined in equation (12) is non-positive, $G^S \leq 0$. Then $r \equiv 0$ is the unique SM equilibrium, that is, $\boldsymbol{\mathcal{SM}}_R = \{0\}$.*

We now consider the case where the gain from trade G^S is positive with $|\, \mathcal{C}^R \,| \geq \kappa$. We also assume without loss of generality that $\kappa \geq 2$, see Footnote 22.

In the following proposition, we show that the intuition in Example 5 holds more generally and provide conditions under which the coalition of promisers can afford to preempt new rounds of promises and achieve stability.

Specifically, we demonstrate that when $G^S \leq \Delta U_{\widehat{\kappa}}$, all equilibria involve a total transfer of promises of G^S from the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ to the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. Importantly, in all equilibria, the post-transfer individual utilities remain larger for all members of $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ than that of any member of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. Before stating the proposition, we recall that $\mathcal{C}^S \cup \underline{\mathcal{C}}^R = \{1, .., \widehat{\kappa}\}$ and $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R = \{\widehat{\kappa} + 1, .., I\}$.

Proposition SA.3. *[**Equilibrium promises with first order preemption.**] Consider a committee in which the support for the reform is larger than the κ-majority requirement, $|\, \mathcal{C}^R \,| \geq \kappa$ with $\kappa \geq 2$. Assume the gain from trade defined in equation (12) satisfies $0 < G^S \leq \Delta U_{\widehat{\kappa}}$ where $\Delta U_{\widehat{\kappa}}$ is defined in equation (13). The net transfer profile r is an SM equilibrium, $r \in \boldsymbol{\mathcal{SM}}_R$, if and only if*

1. *Members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ are promisors subject to individual rationality constraints while members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ are promisees:*

$$-u_j \leq r_j \leq 0 \leq r_i, \quad \forall i \leq \widehat{\kappa} < j. \tag{SA.1}$$

2. *The post-transfer utilities of members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$ cannot exceed those of members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$*

$$\widehat{\pi}_i(r) \leq \widehat{\pi}_j(r), \quad \forall i \leq \widehat{\kappa} < j. \tag{SA.2}$$

3. *The total transfer induced by* r *is given by*

$$\mathcal{T}_r = G^S = \sum_{i=1}^{\widehat{\kappa}} r_i = -\sum_{j=\widehat{\kappa}+1}^{I} r_j. \tag{SA.3}$$

Proposition SA.3 shows that, under the assumption $|\mathcal{C}^R| \geq \kappa$ and $0 < G^S \leq \Delta U_{\widehat{\kappa}}$, all equilibria require members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ to promise a total transfer of G^S to the members of the coalition $\mathcal{C}^S \cup \underline{\mathcal{C}}^R$. In particular, the critical member k_* from Proposition 4 is given by $k_* = \widehat{\kappa} + 1$.

We now consider the case where $|\ \mathcal{C}^R\ | \geq \kappa$ with $\kappa \geq 2$ and with $0 \leq \Delta U_{\widehat{\kappa}} < G^S$. The following proposition characterizes the SM equilibria in that case.

Proposition SA.4. *[* ***Equilibria with higher-order preemption*** *] Consider a committee in which support for the reform is in excess of* κ*-majority requirement,* $|\ \mathcal{C}^R\ | \geq \kappa$ *with* $\kappa \geq 2$*. Assume the gain from trade defined in equation (12) is not affordable to promise, i.e.,* $\Delta U_{\widehat{\kappa}} < G^S$ *where* $\Delta U_{\widehat{\kappa}}$ *is defined in equation (13). A transfer promises profile* $r \in \mathcal{P}$ *is an SM equilibrium, if and only if*

1. *The critical voter* k_* *from Proposition 4 identifies with the* k_* *defined in equation (14).*

2. *Members of the coalition* $\{k_*, \cdots, I\}$ *are promisers subject to individual rationality constraints*

$$-u_j \leq r_j = -u_j + u_* < 0 \quad \textit{for all} \quad j \geq k_*\,, \tag{SA.4}$$

and experience equal ex post intensities

$$\widehat{\pi}_j(r) = u_* > 0 \quad \textit{for all} \quad j \geq k_*. \tag{SA.5}$$

3. *Members of the coalition* $\{1, \cdots, k_* - 1\}$ *are promisees, and their ex post intensities cannot exceed those of promisers*

$$r_i \geq 0 \quad \textit{and} \quad \widehat{\pi}_i(r) \leq \widehat{\pi}_j(r) \equiv u_*, \quad \textit{for all} \quad i < k_* \leq j. \tag{SA.6}$$

Moreover, equilibrium promises profiles are indeterminate, and they all generate the common total promises transfer

$$\mathcal{T}_r = \mathcal{T}_* \text{ where } \mathcal{T}_* := \sum_{j \geq k_*} \left[u_j - u_*\right] > G^S. \tag{SA.7}$$

Proposition SA.4 shows that in all equilibria, members of the promisers coalition promise a transfer that is affine in their utilities as described in equation (SA.4). This implies that members with larger intensities promise a larger transfer and this results in an equalized distribution of the post-transfer utilities among the coalition of promisers. Therefore, in all equilibria, the distribution of transfer promises among promisers is unique. This uniqueness is novel and contrasts with the cases covered in Proposition SA.1 and Proposition SA.3 where the distribution of transfer promises among promisers was indeterminate. The intuition of this uniqueness is that if the promisers have unequal post-transfer utilities, some of them will become subject to enticement to cast their vote against the reform and this, in turn, contradicts the strong Nash requirement as additional promises are required to preclude additional enticements. On the other hand, the multiplicity associated with the transfer distribution among promisees remains valid as in the cases covered in Proposition SA.3.

Proposition SA.4 also shows that the total transfer is larger than the gain from trade G^S as we illustrated in Example 6. More specifically, a direct calculation shows that[23] the aggregate promises from the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$

$$\sum_{j \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} r_j = \sum_{j=\widehat{\kappa}+1}^{I} \left[u_j - u_*\right] = G^S. \tag{SA.8}$$

This shows that members of the coalition $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ will promise an aggregate transfer of G^S. An aggregate transfer of G^S will be sufficient to achieve an equilibrium when

[23] Indeed, by (14), the fact that $\hat{\kappa} = I - \kappa + 1$, and (12), we have

$$\sum_{j=\widehat{\kappa}+1}^{I} \left[u_j - u_*\right] = \sum_{j=\widehat{\kappa}+1}^{I} u_j - (I - \widehat{\kappa})u_* = \sum_{j=\widehat{\kappa}+1}^{I} u_j - (\kappa - 1)u_* = \sum_{j=\widehat{\kappa}+1}^{I} u_j - \sum_{j=1}^{I} u_j = -\sum_{j=1}^{\widehat{\kappa}} u_j = G^S.$$

the gain from trade is affordable. However, this is not the case since $\Delta U_{\widehat{\kappa}} < G^S$ and as a result, some members of $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$ will have lower post-transfer utilities than some members of $\underline{\mathcal{C}}^R$ and hence become targets for enticement. To preempt this from happening, additional transfer promises are needed from the members of the coalition $\{k_*, .., \widehat{\kappa}\} \subseteq \underline{\mathcal{C}}^R$ with *interim* intensities that are larger than those of some members of $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R$. The aggregation of these promises are given by $\sum_{j=k_*}^{\widehat{\kappa}} \left[u_j - u_*\right] > 0$, which results in a total transfer $\mathcal{T}_* > G^S$.

SA.3 Proofs of the propositions in the Supplemental Appendix A

Proof of Proposition SA.1: We start with the following lemma.

Lemma SA.1. *For any promises profile $r \in \mathcal{P}$, the following holds:*

(i) For any coalition $\mathcal{C} \subset \mathbb{I}$, we have $\mathcal{T}_r \geq \big| \sum_{i \in \mathcal{C}} r_i \big|$.

(ii) Consider a coalition $\mathcal{C} \subset \mathbb{I}$ such that $|\mathcal{C}| = \widehat{\kappa}$, $u_i + r_i \leq u_j + r_j$ for all $i \in \mathcal{C}$ and $j \notin \mathcal{C}$. Then $r \in \boldsymbol{\mathcal{S}}_R$ if and only if $\sum_{i \in \mathcal{C}} (u_i + r_i) \geq 0$.

Proof. We first prove the statement (i). Since $r \in \mathcal{P}$, we have

$$\begin{aligned} \mathcal{T}_{\boldsymbol{r}} &= \frac{1}{2}\Big[\sum_{i\in\mathcal{C}} |r_i| + \sum_{i\notin\mathcal{C}} |r_i|\Big] \geq \frac{1}{2}\Big[\big|\sum_{i\in\mathcal{C}} r_i\big| + \big|\sum_{i\notin\mathcal{C}} r_i\big|\Big] \\ &= \frac{1}{2}\Big[\big|\sum_{i\in\mathcal{C}} r_i\big| + \big|-\sum_{i\in\mathcal{C}} r_i\big|\Big] = \big|\sum_{i\in\mathcal{C}} r_i\big|. \end{aligned}$$

We next prove the statement (ii). First, if $r \in \boldsymbol{\mathcal{S}}_R$, since $|\mathcal{C}| = \widehat{\kappa}$, by (16) we have $\sum_{i\in\mathcal{C}} (u_i + r_i) \geq 0$.

We now assume $\sum_{i\in\mathcal{C}} (u_i + r_i) \geq 0$ for $\mathcal{C}$ satisfying the conditions in part (ii) of Lemma SA.1 and prove that $r \in \boldsymbol{\mathcal{S}}_R$. Condition (ii) of Lemma SA.1 implies that

$$\min_{j\notin\mathcal{C}} (u_j + r_j) \geq \max_{i\in\mathcal{C}} (u_i + r_i) \geq 0. \tag{SA.9}$$

For any coalition $\tilde{\mathcal{C}}$ satisfying $|\tilde{\mathcal{C}}| \geq \hat{\kappa}$, consider the partition of $\tilde{\mathcal{C}}$ defined by $\tilde{\mathcal{C}} = \mathcal{C}_1 \cup \mathcal{C}_2 \cup \mathcal{C}_3$. Members of the coalition $\mathcal{C}_1$ belong both to $\mathcal{C}$ and $\tilde{\mathcal{C}}$, that is, $\mathcal{C}_1 := \mathcal{C} \cap \tilde{\mathcal{C}}$. The coalition $\mathcal{C}_2$ is a subset of $\tilde{\mathcal{C}} \backslash \mathcal{C}$ such that when merged with the coalition $\mathcal{C}_1$, it forms a coalition with cardinality $\widehat{\kappa}$, that is, $|\mathcal{C}_1| + |\mathcal{C}_2| = \widehat{\kappa}$. Finally, the coalition $\mathcal{C}_3$ formed by the residual members of $\tilde{\mathcal{C}}$ who do not belong to $\mathcal{C}_1$ or $\mathcal{C}_2$, that is, $\mathcal{C}_3 := \tilde{\mathcal{C}} \backslash (\mathcal{C}_1 \cup \mathcal{C}_2)$. Note that $u_i + r_i \leq u_j + r_j$ for all $i \in \mathcal{C} \backslash \mathcal{C}_1$ and $j \in \mathcal{C}_2$ and that, $|\mathcal{C} \backslash \mathcal{C}_1| = |\mathcal{C}_2|$. Thus, the inequality (SA.9) implies

$$\sum_{j \in \mathcal{C}_2} (u_j + r_j) \geq \sum_{i \in \mathcal{C} \backslash \mathcal{C}_1} (u_i + r_i). \tag{SA.10}$$

Since $\mathcal{C}_3 \cap \mathcal{C} = \emptyset$, we have $u_k + r_k \geq \max_{i \in \mathcal{C}} (u_i + r_i) \geq 0$ for all $k \in \mathcal{C}_3$, and hence $\sum_{k \in \mathcal{C}_3} (u_k + r_k) \geq 0$. Using this last inequality and (SA.10) yields

$$\begin{aligned} \sum_{i \in \tilde{\mathcal{C}}} (u_i + r_i) &= \sum_{i \in \mathcal{C}_1} (u_i + r_i) + \sum_{j \in \mathcal{C}_2} (u_j + r_j) + \sum_{k \in \mathcal{C}_3} (u_k + r_k) \\ &\geq \sum_{i \in \mathcal{C}_1} (u_i + r_i) + \sum_{i \in \mathcal{C} \backslash \mathcal{C}_1} (u_i + r_i) + 0 = \sum_{i \in \mathcal{C}} (u_i + r_i) \geq 0. \end{aligned}$$

Now it follows from (16) again that $r \in \boldsymbol{\mathcal{S}}_R$. □

We now prove Proposition SA.1. We first show that $\mathcal{T}_r \geq U^S$ for any net transfer $r \in \boldsymbol{\mathcal{S}}_R$. Indeed, since $|\mathcal{C}^R| < \kappa$, we have $|\mathcal{C}^S| \geq \widehat{\kappa}$, then (16) implies that $\sum_{\mathcal{C}^S} (u_i + r_i) \geq 0$. Therefore,

$$\sum_{i \in \mathcal{C}^S} |r_i| \geq \sum_{i \in \mathcal{C}^S} r_i \geq \sum_{i \in \mathcal{C}^S} (-u_i) = U^S. \tag{SA.11}$$

Then it follows from Lemma SA.1 (i) that

$$\mathcal{T}_r \geq |\sum_{i \in \mathcal{C}^S} r_i| \geq U^S, \text{ for all } r \in \boldsymbol{\mathcal{S}}_R. \tag{SA.12}$$

We prove the equivalence in Proposition SA.1 by considering two cases separately.

Case 1: $|\mathcal{C}^R| < \kappa - 1$. We cover first the "if" direction ($\Leftarrow$), then the "only if" direction ($\Rightarrow$).

If a promises profile r satisfies conditions 1., 2. and 3.a from Proposition SA.1, then it can be directly checked that $r \in \boldsymbol{\mathcal{P}}$ and $\mathcal{T}_r = U^S$. Moreover, by construction we see that $u_i + r_i = 0$ for $i \in \mathcal{C}^S$ and $u_j + r_j \geq 0$ for $j \in \mathcal{C}^R$. Therefore, $\sum_{i \in \mathcal{C}}(u_i + r_i) \geq 0$ for all coalitions $|\mathcal{C}| \geq \widehat{\kappa}$ and hence, by (16) the promises profile r is a strong Nash equilibrium, that is $r \in \boldsymbol{\mathcal{S}}_R$. Since $\mathcal{T}_r = U^S$, inequality (SA.12) shows that the promises profile r achieves the minimum total promises transfer, and thus $r \in \boldsymbol{\mathcal{SM}}_R$.[24]

We now prove the only if part. That is, we assume $r \in \boldsymbol{\mathcal{SM}}_R$ and prove that conditions 1., 2. and 3.a from Proposition SA.1 hold. Note that

$$\sum_{i \in \mathcal{C}^R} |r_i| \geq \sum_{i \in \mathcal{C}^R} (-r_i) = \sum_{i \in \mathcal{C}^S} r_i \geq U^S, \tag{SA.14}$$

and combining (SA.11) with (SA.14) gives

$$\mathcal{T}_r = \frac{1}{2} \sum_{i \in \mathcal{C}^R} |r_i| + \frac{1}{2} \sum_{i \in \mathcal{C}^S} |r_i| \geq \frac{1}{2} \sum_{i \in \mathcal{C}^R} (-r_i) + \frac{1}{2} \sum_{i \in \mathcal{C}^S} (r_i) \geq U^S. \tag{SA.15}$$

Since $r \in \boldsymbol{\mathcal{SM}}_R$ has minimum total promises transfer, we must have $\mathcal{T}_r = U^S$. Therefore, inequalities (SA.15) become equalities, that is,

$$U^S = \mathcal{T}_r = \frac{1}{2} \sum_{i \in \mathcal{C}^R} |r_i| + \frac{1}{2} \sum_{i \in \mathcal{C}^S} |r_i| = \frac{1}{2} \sum_{i \in \mathcal{C}^R} (-r_i) + \frac{1}{2} \sum_{i \in \mathcal{C}^S} (r_i) = U^S.$$

This in turn implies that inequalities (SA.11) and (SA.14) are also equalities:

$$\sum_{i \in \mathcal{C}^S} |r_i| = \sum_{i \in \mathcal{C}^S} r_i = U^S, \quad \sum_{i \in \mathcal{C}^R} |r_i| = \sum_{i \in \mathcal{C}^R} (-r_i) = U^S.$$

Then

$$|r_i| = r_i, \ i \in \mathcal{C}^S; \quad |r_i| = -r_i, \ i \in \mathcal{C}^R; \quad \text{and} \quad \sum_{i \in \mathcal{C}^S} r_i = \sum_{i \in \mathcal{C}^R} |r_i| = U^S,$$

[24] In particular, we note that the following promises profile r satisfies 1., 2. and 3.a from Proposition SA.1, and hence is an equilibrium with minimum total promises transfer:

$$r_i := -u_i, \ i \in \mathcal{C}^S; \qquad r_j := -\frac{U^S}{U^R} u_j, \ j \in \mathcal{C}^R. \tag{SA.13}$$

and thus

$$r_i \geq 0 \geq r_j, \quad \forall i \in \mathcal{C}^S, j \in \mathcal{C}^R, \quad \text{and} \quad \sum_{i \in \mathcal{C}^S} r_i = U^S = \sum_{j \in \mathcal{C}^R} (-r_j). \tag{SA.16}$$

Moreover, the equality $\sum_{i \in \mathcal{C}^S} r_i = U^S$ implies that $\sum_{i \in \mathcal{C}^S}(u_i + r_i) = 0$.

Since $|\mathcal{C}^S| > \widehat{\kappa}$, then for any $i_0 \in \mathcal{C}^S$, $|\mathcal{C}^S \backslash \{i_0\}| \geq \widehat{\kappa}$, thus by (16) we have $\sum_{i \in \mathcal{C}^S \backslash \{i_0\}}(u_i + r_i) \geq 0$. This, together with $\sum_{i \in \mathcal{C}^S}(u_i + r_i) = 0$, implies that $u_{i_0} + r_{i_0} \leq 0$, for all $i_0 \in \mathcal{C}^S$. Combining this with the equality $\sum_{i \in \mathcal{C}^S}(u_i + r_i) = 0$, we must have $u_i + r_i = 0$ or $r_i = -u_i$ for all $i \in \mathcal{C}^S$.

Finally, for each $j \in \mathcal{C}^R$, since $|\mathcal{C}^S \cup \{j\}| \geq \widehat{\kappa}$, then by (16) we have $0 \leq \sum_{i \in \mathcal{C}^S \cup \{j\}}(u_i + r_i) = u_j + r_j$. That is, $r_j \geq -u_j$ for all $j \in \mathcal{C}^R$. To summarize, we have proven that, in addition to the relations (SA.16), $r_i = -u_i$ for all $i \in \mathcal{C}^S$ and $-u_j \leq r_j \leq 0$ for all $j \in \mathcal{C}^R$. Thus, conditions 1 and 2.a from Proposition SA.1 hold. This concludes the proof for the case $|\mathcal{C}^R| < \kappa - 1$.

Case 2: $|\mathcal{C}^R| = \kappa - 1$. Notice that in this case, $|\mathcal{C}^S| = \widehat{\kappa}$. First, if r satisfies conditions 1 and 2.b of Proposition SA.1, then it can be checked as in Step 1 that $r \in \mathcal{P}$, $\mathcal{T}_r = U^S$, $\sum_{i \in \mathcal{C}^S}(u_i + r_i) = 0$, and $r_j \geq -u_j$ for all $j \in \mathcal{C}^R$. Moreover, by condition 2.b of Proposition SA.1 and property (ii) of Lemma SA.1, we see that $r \in \boldsymbol{\mathcal{S}}_R$. Since $\mathcal{T}_r = U^S$, the net transfer r minimizes the total promises transfer and hence $r \in \boldsymbol{\mathcal{SM}}_R$.[25]

We now prove the only if part. We assume $r \in \boldsymbol{\mathcal{SM}}_R$ and show that conditions 1., 2. and 3.b of Proposition SA.1 hold true. By the same arguments in Step 1 of this proof, we see that (SA.16) still holds true. Moreover, for any $i_0 \in \mathcal{C}^S$ and $j_0 \in \mathcal{C}^R$,

[25]In particular, we note that the promises profile r constructed in (SA.13) satisfies conditions 1 and 2.b of of Proposition SA.1 and hence it belongs to $\boldsymbol{\mathcal{SM}}_R$. In contrast to the case where $|\mathcal{C}^S| < \kappa - 1$ where all members of the coalition $\mathcal{C}^S$ become indifferent between R and S after receiving the equilibrium transfer promises, we note that in the case $|\mathcal{C}^R| = \kappa - 1$ it is possible that $u_i + r_i > 0$ for some $i \in \mathcal{C}^S$.

note that $|(\mathcal{C}^S \backslash \{i_0\}) \cup \{j_0\}| \geq \widehat{\kappa}$, then by (16) we have

$$
\begin{aligned}
0 \leq \sum_{i \in (\mathcal{C}^S \backslash \{i_0\}) \cup \{j_0\}} (u_i + r_i) &= \sum_{i \in \mathcal{C}^S} (u_i + r_i) - (u_{i_0} + r_{i_0}) + (u_{j_0} + r_{j_0}) \\
&= (u_{j_0} + r_{j_0}) - (u_{i_0} + r_{i_0}).
\end{aligned}
$$

Thus, $u_{i_0} + r_{i_0} \leq u_{j_0} + r_{j_0}$ for all $i_0 \in \mathcal{C}^S$ and $j_0 \in \mathcal{C}^R$ and hence, Condition 3.(b) of Proposition SA.1 is satisfied. With the exception of the inequality $-u_j \leq r_j$ for $j \in \mathcal{C}^R$, all other properties in Condition 1. and 2. from Proposition SA.1 are implied by (SA.16). To prove this last property, note that if $\sum_{i \in \mathcal{C}^S}(u_i + r_i) = 0$, then there exists $i_0 \in \mathcal{C}^S$ such that $u_{i_0} + r_{i_0} \geq 0$. Condition 2. from Proposition SA.1 implies that $u_j + r_j \geq u_{i_0} + r_{i_0} \geq 0$ for any $j \in \mathcal{C}^R$, and thus $r_j \geq -u_j$, for all $j \in \mathcal{C}^R$. □

Proof of Proposition SA.2: Note that in this case $\sum_{i=1}^{\widehat{\kappa}} u_i = -G^S \geq 0$. It can be verified that the conditions in Lemma SA.1 (ii) for $\mathcal{C} = \{1, \cdots, \widehat{\kappa}\}$ and $r = 0$ hold. Then $0 \in \boldsymbol{\mathcal{S}}_R$. This degenerate transfer r^{t^0} has zero total transfers and hence it is unique, $\boldsymbol{\mathcal{SM}}_R = \{\mathbf{0}\}$. □

Proof of Proposition SA.3: The proof proceeds in four steps. In the first step, we show that $\mathcal{T}_r \geq G^S$ for all $r \in \boldsymbol{\mathcal{S}}_R$. In the second step, we prove the "if" part of Proposition SA.3. In the third step, we give an example of across the aisle equilibrium promise that satisfies $\mathcal{T}_r = G^S$. The example is used in the fourth and last step where we prove the "only if" part of Proposition SA.3.

Step 1. We first show that $\mathcal{T}_r \geq G^S$ for any $r \in \boldsymbol{\mathcal{S}}_R$. Since $|\mathcal{C}^S \cup \underline{\mathcal{C}}^R| \geq \widehat{\kappa}$, by (12) and (16) we have

$$
0 \leq \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} [u_i + r_i] = -G^S + \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i. \qquad \text{(SA.17)}
$$

Then by Lemma SA.1 (i) we have $\mathcal{T}_r \geq |\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i| \geq \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i \geq G^S$.

Step 2. We next prove the if part: assume conditions (SA.1), (SA.2) and (SA.3) hold and prove that $r \in \boldsymbol{\mathcal{SM}}_R$. Let $r \in \mathcal{P}$ satisfy (SA.1), note that $r_k \geq -u_k$ or $u_k + r_k \geq 0$ for all $k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R$, and by the calculation in (SA.17) we see that

$\sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R}\widehat{\pi}_i(r)\geq 0$. These observations, together with condition (SA.2) show that all conditions required in Lemma SA.1 (ii) are fulfilled, and therefore we have $r\in\boldsymbol{\mathcal{S}}_R$. Condition (SA.3) and Step 1 of this proof show that in addition to being a strong Nash equilibrium, the net transfer r achieves the minimal total promises transfer $\mathcal{T}_r = G^S$, and thus $r\in\boldsymbol{\mathcal{SM}}_R$.

Step 3. In this step we construct an equilibrium $r\in\boldsymbol{\mathcal{SM}}_R$. Consider the following across the aisle net transfer r:

$$r_i := -\frac{G^S}{U^S}u_i,\ i\in\mathcal{C}^S;\quad r_j := 0,\ j\in\underline{\mathcal{C}}^R;\quad r_k := -\frac{G^S}{\Delta U_{\widehat{\kappa}}}[u_k-u_{\widehat{\kappa}}],\ k\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R. \tag{SA.18}$$

We can show directly that it satisfies conditions (SA.1), (SA.2) and, (SA.3).[26] Then, using the result in Step 2 of this proof shows that the promises profile r is an SM equilibrium.

Step 4. We now prove the only if part. Let $r\in\boldsymbol{\mathcal{SM}}_R$ and prove that r satisfies conditions (SA.1), (SA.2) and, (SA.3). Applying Step 1 of this proof shows that, since

[26] To see this, first observe that $r\in\mathcal{P}$ since, using (10) and (13), it can be checked that $\sum_{\mathbb{I}} r_i = 0$. Noticing that $u_{\widehat{\kappa}}\geq 0$, $G^S\leq U^S$, and $G^S\leq\Delta U_{\widehat{\kappa}}$, we have for any $i\in\mathcal{C}^S$, $j\in\underline{\mathcal{C}}^R$, and $k\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R$,

$$r_i\geq 0,\quad r_j=0,\quad r_k\leq 0;\qquad r_k\geq -\frac{G_s}{U_{\widehat{\kappa}}}u_k\geq -u_k\,,$$

and therefore r satisfies condition (SA.1). Moreover, we have,

$$\widehat{\pi}_i(r) = [1-\tfrac{G^S}{U_s}]u_i < 0\leq u_{\widehat{\kappa}},\quad \widehat{\pi}_j(r)=u_j\leq u_{\widehat{\kappa}},$$
$$\widehat{\pi}_k(r) = u_k - \tfrac{G_s}{\Delta U_{\widehat{\kappa}}}[u_k-u_{\widehat{\kappa}}]\geq u_k-[u_k-u_{\widehat{\kappa}}]=u_{\widehat{\kappa}};$$

and thus condition (SA.2) is satisfied. Finally,

$$\textstyle\sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R} r_i = \frac{G^S}{U^S}\sum_{i\in\mathcal{C}^S}[-u_i] = G^S,$$
$$\textstyle\sum_{k\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R} r_k = -\frac{G_s}{\Delta U_{\widehat{\kappa}}}\sum_{k\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R}[u_k-u_{\widehat{\kappa}}] = -\frac{G_s}{\Delta U_{\widehat{\kappa}}}\Delta U_{\widehat{\kappa}} = -G^S;$$

and hence condition (SA.3) is satisfied. To sum up, the net transfer r satisfy conditions (SA.1), (SA.2) and, (SA.3), and therefore it is an SM equilibrium.

$r \in \boldsymbol{\mathcal{S}}_R$, we have $\mathcal{T}_r \geq G^S$. Recall (SA.17) and note that

$$\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} |r_i| \geq \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i \geq G^S; \tag{SA.19}$$

$$\sum_{k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} |r_k| \geq \sum_{k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} (-r_k) = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i \geq G^S. \tag{SA.20}$$

Combining (SA.19) and (SA.20) gives

$$\mathcal{T}_r = \frac{1}{2} \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} |r_i| + \frac{1}{2} \sum_{k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} |r_k| \geq \frac{1}{2} \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i + \frac{1}{2} \sum_{k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} (-r_k) \geq G^S. \tag{SA.21}$$

Since $\mathcal{T}_r \geq G^S$ for any equilibrium net transfer, and since the equilibrium net transfer defined in (SA.18) achieves the total promises transfer G^S, it must be that $\mathcal{T}_r = G^S$ for any $r \in \boldsymbol{\mathcal{SM}}_R$. When $\mathcal{T}_r = G^S$, inequalities (SA.21) become equalities:

$$G^S = \mathcal{T}_r = \frac{1}{2} \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} |r_i| + \frac{1}{2} \sum_{k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} |r_k| = \frac{1}{2} \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i + \frac{1}{2} \sum_{k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} (-r_k) = G^S.$$

This, in turn, implies inequalities (SA.19) and (SA.20) are also equalities:

$$\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} |r_i| = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i = G^S, \qquad \sum_{j \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} |r_j| = \sum_{j \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} (-r_j) = G^S.$$

Then similarly to the approach used to prove (SA.16), we deduce that $|r_i| = r_i$ for all $i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R$ and $|r_j| = -r_j$ for all $j \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R$. Hence

$$r_j \leq 0 \leq r_i, \quad \forall i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R, j \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R.$$

Recalling that $\mathcal{C}^S \cup \underline{\mathcal{C}}^R = \{1, .., \widehat{\kappa}\}$ and $\mathcal{C}^R \backslash \underline{\mathcal{C}}^R = \{\widehat{\kappa} + 1, .., I\}$, we see that the last equation is equivalent to

$$r_j \leq 0 \leq r_i, \quad \forall i \leq \hat{\kappa} < j.$$

Moreover,

$$\mathcal{T}_r = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} r_i \equiv \sum_{i=1}^{\widehat{\kappa}} r_i = \sum_{j \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R} (-r_j) \equiv \sum_{j=\widehat{\kappa}+1}^{I} (-r_j) = G^S.$$

Using (12), observe further that

$$\sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R}(u_i+r_i)=\sum_{i\in\mathcal{C}^S}u_i+\sum_{i\in\underline{\mathcal{C}}^R}u_i+\sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R}r_i=-U^S+\underline{U}^R+G^S=0.$$

To verify the individual rationality constraint $-u_j\le r_j$ for any $j\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R$, observe that $|(\mathcal{C}^S\cup\underline{\mathcal{C}}^R)\cup\{j\}|\ge\widehat{\kappa}n$, then by (16) we have

$$0\le\sum_{i\in(\mathcal{C}^S\cup\underline{\mathcal{C}}^R)\cup\{j\}}(u_i+r_i)=\sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R}(u_i+r_i)+(u_j+r_j)=u_j+r_j,$$

which implies $r_j\ge-u_j$. So far, we have shown that the net transfer r satisfies (SA.1) and (SA.3).

All that's left to prove is that (SA.2) also holds. For any $j\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R$ and $k\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R$, note that $|(\mathcal{C}^S\cup\underline{\mathcal{C}}^R\cup\{k\})\backslash\{j\}|=\widehat{\kappa}$, then by (16) we have

$$0\le\sum_{i\in(\mathcal{C}^S\cup\underline{\mathcal{C}}^R\cup\{k\})\backslash\{j\}}(u_i+r_i)=\sum_{i\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R}(u_i+r_i)+(u_k+r_k)-(u_j+r_j)=\widehat{\pi}_k(r)-\widehat{\pi}_j(r).$$

Thus, $\widehat{\pi}_j(r)\le\widehat{\pi}_k(r)$ for all $j\in\mathcal{C}^S\cup\underline{\mathcal{C}}^R$ and $k\in\mathcal{C}^R\backslash\underline{\mathcal{C}}^R$, and the net transfer r satisfies (SA.2), which completes Step 4 and thus concludes the proof. □

Proof of Proposition SA.4: We start the proof with two preliminary lemmas.

Lemma SA.2. *Consider a committee operating under the κ-majority rule with $|\kappa|\ge2$ and such that $G^S>\Delta U_{\widehat{\kappa}}$. The following statements hold:*

(i) Recalling the utility u_* defined in (14)*, we have*

$$u_*<u_{\widehat{\kappa}}. \tag{SA.22}$$

(ii) There exists a unique group member $k_*\in\underline{\mathcal{C}}^R$, i.e., $n<\kappa_*\le\widehat{\kappa}$, such that inequalities (14) *hold, that is, $u_{k_*-1}\le u_*<u_{k_*}$.*

(iii) The constant $\mathcal{T}_*$ defined in (SA.7) *satisfies the inequality*

$$\mathcal{T}_*>G^S. \tag{SA.23}$$

Proof. To prove that (i) holds, observe that

$$\begin{aligned} G^S - \Delta U_{\widehat{\kappa}} &= \sum_{i=\widehat{\kappa}+1}^{I} [u_i - u_*] - \sum_{i=\widehat{\kappa}+1}^{I} [u_i - u_{\widehat{\kappa}}] \\ &= (I - \hat{\kappa})[u_{\widehat{\kappa}} - u_*] = (\kappa - 1)[u_{\widehat{\kappa}} - u_*]. \end{aligned} \tag{SA.24}$$

where we have used (SA.8), the definition of $\Delta U_{\widehat{\kappa}}$ in (13), and the relation $\widehat{\kappa} = I - \kappa + 1$. Since $G^S > \Delta U_{\hat{\kappa}}$, this establishes inequality (SA.22) in part (i) of Lemma SA.2.

By the relations (14), and (SA.22), we have $u_n < 0 < u_* < u_{\widehat{\kappa}}$. Recalling that the utilities u_i are ordered, there exists a unique $k_* \in \underline{\mathcal{C}}^R$, i.e., $n < k_* \leq \widehat{\kappa}$, such that the inequalities (14) hold. Indeed, $k_* := \min k$ such that $u_k > u_*$. This proves part (ii) of Lemma SA.2.

Finally, observe that the total promises transfer $\mathcal{T}_*$ defined in (SA.7) satisfies

$$\mathcal{T}_* \equiv \sum_{j=k_*}^{\widehat{\kappa}} [u_j - u_*] + \sum_{j=\widehat{\kappa}+1}^{I} [u_j - u_*] > \sum_{j=\widehat{\kappa}+1}^{I} [u_j - u_*] = G^S, \tag{SA.25}$$

where the first inequality is due to (14), and the last equality is due to (SA.8). This establishes inequality (SA.23) in part (iii) of Lemma SA.2. □

Lemma SA.3. *Consider the across the aisle net transfer profile r defined by*

$$r_i := -\frac{\mathcal{T}_*}{U^S} u_i, \ \forall i \leq n; \quad r_k := 0, \ \forall n < k < k_*; \quad r_j := -[u_j - u_*], \ \forall j \geq k_*, \tag{SA.26}$$

where we recall that k_* is defined in part (ii) of Lemma SA.2. Then the net transfer r is zero sum, $r \in \mathcal{P}$ and satisfies the conditions (SA.4)-(SA.7) *of Proposition SA.4. Moreover, the net transfer profile r defined in* (SA.26) *is a strong equilibrium.*

Proof. First, $r \in \mathcal{P}$ since

$$\sum_{i \in \mathbb{I}} r_i = \frac{\mathcal{T}_*}{U^S} \sum_{i=1}^{n} (-u_i) - \sum_{j=k_*}^{I} [u_j - u_*] = \mathcal{T}_* - \mathcal{T}_* = 0.$$

Second, the statements in (SA.4), (SA.5) and (SA.7) can be directly checked from the definition of the net transfer r given in (SA.26).

Next, to prove (SA.6), we need first to prove the preliminary result that $0 < \mathcal{T}_* < U^S$. Using (SA.25), observe that

$$U^S - \mathcal{T}_* = U^S - \sum_{j=k_*}^{\widehat{\kappa}} [u_j - u_*] - G^S.$$

Using the definition of G^S given in (12) yields

$$U^S - \mathcal{T}_* = \underline{U}^R - \sum_{j=k_*}^{\widehat{\kappa}} [u_j - u_*] = \underline{U}^R - \sum_{j=k_*}^{\widehat{\kappa}} u_j + (\widehat{\kappa} - k_* + 1)u_*. \tag{SA.27}$$

Since $k_* \in \underline{\mathcal{C}}^R$, we have $n < k_* \leq \widehat{\kappa}$ and hence $\underline{U}^R - \sum_{j=k_*}^{\widehat{\kappa}} u_j \geq 0$. Moreover, $n < k_* \leq \widehat{\kappa}$ also implies that $(\widehat{\kappa} - k_* + 1)u_* > 0$. Thus, using (SA.27) gives $0 < \mathcal{T}_* < U^S$.

Now we are prepared to prove (SA.6). For $i \leq n$, $n < k < k_*$, and $j \geq k_*$, we have

$$\widehat{\pi}_i(r) = u_i + r_i = [1 - \tfrac{\mathcal{T}_*}{U^S}]u_i \leq 0;$$
$$\widehat{\pi}_k(r) = u_k + r_k = u_k \geq 0; \quad \widehat{\pi}_j(r) = u_j + r_j = u_* > 0,$$

where the first inequality is implied by $0 < \mathcal{T}_* < U^S$. Thus, we see that the condition (SA.6) is satisfied for the net transfer profile r defined in (SA.26).

Finally, to show that r is a strong equilibrium, set $\mathcal{C} := \mathcal{C}^S \cup \underline{\mathcal{C}}^R$ in Lemma SA.1 (ii) so that $|\mathcal{C}| = \widehat{\kappa}$. Using condition (SA.6) and observing that $u_j + r_j = u_*$ for all $j \geq k_*$ shows that we have $u_i + r_i \leq u_j + r_j$ for all $i \in \mathcal{C}$ and $j \notin \mathcal{C}$.

Moreover, since $r \in \mathcal{P}$, by (SA.22) we have

$$\begin{aligned} \sum_{i \in \mathcal{C}} (u_i + r_i) &= \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i - \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} r_j = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i - \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} [-u_j + u_*] \\ &= \sum_{i \in \mathbb{I}} u_i - (\kappa - 1)u_* = 0. \end{aligned}$$

Then by applying Lemma SA.1 (ii), we see that $r \in \boldsymbol{\mathcal{S}}_R$. □

Proof of Proposition SA.4: We proceed in two steps.

Step 1. In this step, we prove the only if part: we fix an equilibrium net transfer profile $r \in \boldsymbol{\mathcal{S}}_R$, and show that r satisfies conditions (SA.4)-(SA.7) of Proposition SA.4.

Notice that Proposition 3 shows that the set $\boldsymbol{\mathcal{S}}_R$ is not empty, and thus it is possible to select a net transfer profile from $\boldsymbol{\mathcal{S}}_R$.

Recall Lemma 3 and let $\tilde{k}_* \in \mathcal{C}^R$ satisfy

$$\underline{k}_* \equiv \max\{i : r_i > 0\} < \tilde{k}_* \leq \min\{i : r_i < 0\} \equiv \overline{k}_*, \tag{SA.28}$$

and the requirements (8) (In this proof, we reserve the notation k_* for the requirement (14) instead of the requirements (8)), that is, $-u_j \leq r_j \leq 0 \leq r_i$ and $\widehat{\pi}_i(r) \leq \widehat{\pi}_j(r)$ for all $i < \tilde{k}_* \leq j$. Moreover, we assume $\tilde{k}_*$ is the largest one satisfying the requirements, and thus

$$\text{either } r_{\tilde{k}_*} < 0, \quad \text{or} \quad \widehat{\pi}_{\tilde{k}_*}(r) > \min_{j > \tilde{k}_*}, \widehat{\pi}_j(r) \tag{SA.29}$$

because otherwise $\tilde{k}_* + 1$ would also satisfy the desired requirements[27].

Step 1.1. We first show that $\tilde{k}_* \leq k_*$. Since $k_* \leq \widehat{\kappa}$, this implies $\tilde{k}_* \leq \widehat{\kappa}$. Assume by contradiction that $\tilde{k}_* > k_*$. Then by (8) we have

$$u_j + r_j \geq u_{k_*} + r_{k_*} \geq u_{k_*}, \quad \text{for all} \quad j \geq \tilde{k}_*.$$

Moreover, in the case $\tilde{k}_* > \widehat{\kappa}$, recalling that the utilities u_i are ordered, we also have,

$$u_j + r_j \geq u_j \geq u_{k_*}, \quad \text{for all} \quad \widehat{\kappa} \leq j < \tilde{k}_*.$$

So in all the cases we have $u_j + r_j \geq u_{k_*}$ for all $j \geq \widehat{\kappa}$. This, in turn, implies,

$$\sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} r_j \geq \sum_{j > \widehat{\kappa}} [u_{k_*} - u_j] = -\Delta U_{k_*} > -G^S.$$

Thus, since $r \in \mathcal{P}$,

$$\sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} [u_i + r_i] = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i - \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} r_j < -G^S + G^S = 0.$$

[27]To see this, assume the opposite of statement (SA.29) holds, that is, $r_{\tilde{k}_*} \geq 0$ and $\widehat{\pi}_{\tilde{k}_*}(r) \leq \min_{j > \tilde{k}_*} \widehat{\pi}_j(r)$. Condition (8) implies that $r_{\tilde{k}_*} \leq 0$, and hence the assumption $r_{\tilde{k}_*} \geq 0$ implies that $r_{\tilde{k}_*} = 0$. Thus, we have $\underline{k}_* < \tilde{k}_* + 1 \leq \overline{k}_*$ and, $-u_j \leq r_j \leq 0 \leq r_i$ for all $i < \tilde{k}_* + 1 \leq j$. Finally, the assumption $\widehat{\pi}_{\tilde{k}_*}(r) \leq \min_{j > \tilde{k}_*} \widehat{\pi}_j(r)$ implies $\widehat{\pi}_i(r) \leq \widehat{\pi}_j(r)$ for all $i < \tilde{k}_* + 1 \leq j$. Hence $\tilde{k}_* + 1$ also satisfies the requirements (SA.28) and (8).

This contradicts (16), which is required here because $r \in \boldsymbol{\mathcal{S}}_R$ and $|\mathcal{C}^S \cup \underline{\mathcal{C}}^R| = \widehat{\kappa}$.

Step 1.2. In this step, we prove that the post-transfer utilities of the members $j = \tilde{k}_*, \cdots, I$ are equal:

$$\widehat{\pi}_{\tilde{k}_*}(r) = \cdots = \widehat{\pi}_I(r). \tag{SA.30}$$

Consider for notational convenience, the order statistics of $\{\widehat{\pi}_j(r)\}_{\tilde{k}_* \le j \le I}$:

$$\widehat{\pi}_{l_{\tilde{k}_*}}(r) \le \cdots \le \widehat{\pi}_{l_I}(r),$$

where $\{l_{\tilde{k}_*}, \cdots, l_I\}$ is a permutation of $\{\tilde{k}_*, \cdots, I\}$.

To simplify the exposition, we proceed in two substeps. First, we show that $\widehat{\pi}_{l_{\tilde{k}_*}}(r) = \cdots = \widehat{\pi}_{l_{\widehat{\kappa}+1}}(r)$. Second, we show that the equality $\widehat{\pi}_{l_{\tilde{k}_*}}(r) = \cdots = \widehat{\pi}_I(r)$ is also true.

Step 1.2.1. In this sub-step we show that

$$\widehat{\pi}_{l_{\tilde{k}_*}}(r) = \widehat{\pi}_{l_{\widehat{\kappa}+1}}(r) \quad \text{and hence} \quad \widehat{\pi}_{l_{\tilde{k}_*}}(r) = \cdots = \widehat{\pi}_{l_{\widehat{\kappa}+1}}(r). \tag{SA.31}$$

Notice that we are considering the term $l_{\widehat{\kappa}+1}$, rather than $l_{\widehat{\kappa}}$.

Assume by contradiction that

$$\widehat{\pi}_{l_{\tilde{k}_*}}(r) < \widehat{\pi}_{l_{\widehat{\kappa}+1}}(r). \tag{SA.32}$$

We claim that

$$r_{l_{\tilde{k}_*}} < 0. \tag{SA.33}$$

Indeed, recall (SA.29). In the case $r_{\tilde{k}_*} < 0$, by the definition of order statistics and recalling that the utilities u_i are ordered, we have

$$u_{l_{\tilde{k}_*}} + r_{l_{\tilde{k}_*}} \le u_{\tilde{k}_*} + r_{\tilde{k}_*} < u_{\tilde{k}_*} \le u_{l_{\tilde{k}_*}},$$

which implies (SA.33). In the case $r_{\tilde{k}_*} = 0$ and $\widehat{\pi}_{\tilde{k}_*}(r) > \min_{j > \tilde{k}_*} \widehat{\pi}_j(r)$, by the ordering of the utilities u_i again, we have

$$u_{\tilde{k}_*} = \widehat{\pi}_{\tilde{k}_*}(r) > \min_{j \ge \tilde{k}_*} \widehat{\pi}_j(r) = \widehat{\pi}_{l_{\tilde{k}_*}}(r) = u_{l_{\tilde{k}_*}} + r_{l_{\tilde{k}_*}} \ge u_{\tilde{k}_*} + r_{l_{\tilde{k}_*}},$$

implying (SA.33) again.

Clearly $r_i > 0$ for some $i < \tilde{k}_*$, and assume without loss of generality that $r_1 > 0$. We now modify r as follows: for some $\varepsilon > 0$ small,

$$\tilde{r}_1 = r_1 - \varepsilon > 0, \quad \tilde{r}_{l_{\tilde{k}_*}} = r_{l_{\tilde{k}_*}} + \varepsilon < 0, \quad \text{and} \quad \tilde{r}_i = r_i \quad \text{for all } i \neq 1, l_{\tilde{k}_*}.$$

Set $\tilde{\mathcal{C}} = \{1, \cdots, \tilde{k}_* - 1\} \cup \{l_{\tilde{k}_*}, \cdots, l_{\widehat{\kappa}}\}$ and notice that $|\tilde{\mathcal{C}}| = \widehat{\kappa}$. Note that, for $\varepsilon > 0$ small enough,

$$u_1 + \tilde{r}_1 < u_1 + r_1 \leq \widehat{\pi}_{l_{\widehat{\kappa}+1}}(r); \qquad u_{l_{\tilde{k}_*}} + \tilde{r}_{l_{\tilde{k}_*}} = \widehat{\pi}_{l_{\tilde{k}_*}}(r) + \varepsilon < \widehat{\pi}_{l_{\widehat{\kappa}+1}}(r);$$
$$u_i + \tilde{r}_i = u_i + r_i \leq \widehat{\pi}_{l_{\widehat{\kappa}+1}}(r), \quad i \in \tilde{\mathcal{C}} \backslash \{1, l_{\tilde{k}_*}\}; \quad u_j + \tilde{r}_j = u_j + r_j \geq \widehat{\pi}_{l_{\widehat{\kappa}+1}}(r), \quad j \notin \tilde{\mathcal{C}}.$$

Thus $u_i + \tilde{r}_i \leq \widehat{\pi}_{l_{\widehat{\kappa}+1}}(r) \leq u_j + \tilde{r}_j$ for all $i \in \tilde{\mathcal{C}}$ and $j \notin \tilde{\mathcal{C}}$. Note further that, since $r \in \boldsymbol{\mathcal{SM}}_R \subset \boldsymbol{\mathcal{S}}_R$,

$$\sum_{i \in \tilde{\mathcal{C}}} [u_i + \tilde{r}_i] = \sum_{i \in \tilde{\mathcal{C}}} [u_i + r_i] + \Big[(\tilde{r}_1 - r_1) + (\tilde{r}_{l_{\tilde{k}_*}} - r_{l_{\tilde{k}_*}})\Big] = \sum_{i \in \tilde{\mathcal{C}}} [u_i + r_i] \geq 0.$$

Then by Lemma SA.1 (ii) we have $\tilde{r} \in \boldsymbol{\mathcal{S}}_R$. However, as in the last part of the proof for Lemma 1, we have $\mathcal{T}_{\tilde{r}} = \mathcal{T}_r - \varepsilon < \mathcal{T}_r$. This contradicts the assumption that $r \in \boldsymbol{\mathcal{SM}}_R$ has the minimum total promises transfer. Therefore, (SA.31) holds true.

Step 1.2.2. Now we proceed to prove (SA.30). Assume by contradiction that, for the order statistics in the previous step,

$$\widehat{\pi}_{l_{\tilde{k}_*}}(r) = \cdots = \widehat{\pi}_{l_{k_2}}(r) < \widehat{\pi}_{l_{k_2+1}}(r) \leq \cdots \leq \widehat{\pi}_{l_I}(r), \quad \text{for some } \widehat{\kappa} + 1 \leq k_2 < I.$$

First, by (SA.33) we also have

$$r_{l_j} < 0, \quad \text{for all} \quad j = \tilde{k}_*, \cdots, k_2. \tag{SA.34}$$

Again assume $r_1 > 0$. We then modify r as follows: for $\varepsilon > 0$ small,

$$\hat{r}_1 = r_1 - [\widehat{\kappa} - \tilde{k}_* + 1]\varepsilon > 0, \quad \hat{r}_{l_j} = r_{l_j} + \varepsilon < 0, \; j = \tilde{k}_*, \cdots, k_2;$$
$$\hat{r}_{l_{k_2+1}} = r_{l_{k_2+1}} - [k_2 - \widehat{\kappa}]\varepsilon < 0; \quad \hat{r}_i = r_i \quad \text{for all other } i.$$

One can check that $\boldsymbol{r} \in \mathcal{P}$:

$$\sum_{i\in\mathbb{I}} \hat{r}_i = \sum_{i\in\mathbb{I}} r_i - [\widehat{\kappa} - \tilde{k}_* + 1]\varepsilon + \sum_{j=\tilde{k}_*}^{k_2} \varepsilon - [k_2 - \widehat{\kappa}]\varepsilon = 0.$$

Similarly to Step 1.2.1, we see that, for all $i < \tilde{k}_*$ and $j > k_2 + 1$,

$$u_i + \hat{r}_i \le u_{l_{\tilde{k}_*}} + \hat{r}_{l_{\tilde{k}_*}} = \cdots = u_{l_{k_2}} + \hat{r}_{l_{k_2}} < u_{l_{k_2+1}} + \hat{r}_{l_{k_2+1}} < u_{l_j} + \hat{r}_{l_j},$$

where the second inequality holds for $\varepsilon > 0$ small enough. Now for the same $\tilde{\mathcal{C}} = \{1, \cdots, \tilde{k}_* - 1\} \cup \{l_{\tilde{k}_*}, \cdots, l_{\widehat{\kappa}}\}$ with $|\tilde{\mathcal{C}}| = \widehat{\kappa}$ as in Step 1.2.1, we have

$$\begin{aligned}
&u_i + \hat{r}_i \le u_{l_{\tilde{k}_*}} + \hat{r}_{l_{\tilde{k}_*}} \le u_j + \hat{r}_j, \quad \text{for all } i \in \tilde{\mathcal{C}},\ j \notin \tilde{\mathcal{C}}; \\
&\sum_{i\in\tilde{\mathcal{C}}} [u_i + \hat{r}_i] = \sum_{i\in\tilde{\mathcal{C}}} [u_i + r_i] - [\widehat{\kappa} - \tilde{k}_* + 1]\varepsilon + \sum_{j=\tilde{k}_*}^{\widehat{\kappa}} \varepsilon = \sum_{i\in\tilde{\mathcal{C}}} [u_i + r_i] \ge 0.
\end{aligned}$$

Then by Lemma SA.1 (ii) we see that $\hat{r} \in \boldsymbol{\mathcal{S}}_R$. Moreover, note that

$$\begin{aligned}
\mathcal{T}_{\hat{r}} - \mathcal{T}_r &= \frac{1}{2}\Big[|\hat{r}_1| - |r_1| + \sum_{j=\tilde{k}_*}^{k_2} [|\hat{r}_{l_j}| - |r_{l_j}|] + |\hat{r}_{l_{k_2+1}}| - |r_{l_{k_2+1}}|\Big] \\
&= \frac{1}{2}\Big[- [\widehat{\kappa} - \tilde{k}_* + 1]\varepsilon + \sum_{j=\tilde{k}_*}^{k_2} (-\varepsilon) + [k_2 - \widehat{\kappa}]\varepsilon\Big] = -[\widehat{\kappa} - \tilde{k}_* + 1]\varepsilon < 0,
\end{aligned}$$

where the last inequality is due to $\tilde{k}_* \le \widehat{\kappa}$ from Step 1.1. This contradicts the assumption that $r \in \boldsymbol{\mathcal{SM}}_R$ has the minimum total promises transfer, so equation (SA.30) holds true.

Step 1.3. We now collect all the results from the intermediate steps to show that r satisfies conditions (SA.4)-(SA.7) of Proposition SA.4.

Let y_* denote the common value in (SA.30). Then $r_j = y_* - u_j \le 0$ for all $j \ge \tilde{k}_*$. On one hand, since $\tilde{k}_* \le k_* \le \widehat{\kappa}$ by Step 1.1,

$$\begin{aligned}
0 &\le \sum_{i\le\widehat{\kappa}} [u_i + r_i] = \sum_{i\le\widehat{\kappa}} u_i - \sum_{j>\widehat{\kappa}} r_j = \sum_{i\le\widehat{\kappa}} u_i - \sum_{j>\widehat{\kappa}} [y_* - u_j] \\
&= \sum_{i\in\mathbb{I}} u_i - (\kappa - 1)y_* = (\kappa - 1)(u_* - y_*).
\end{aligned}$$

Therefore, $y_* \le u_*$. On the other hand, by (9),

$$\mathcal{T}_r = \sum_{j \ge \tilde{k}_*} (-r_j) \ge \sum_{j \ge k_*} (-r_j) = \sum_{j \ge k_*} [u_j - y_*] \ge \sum_{j \ge k_*} [u_j - u_*] = \mathcal{T}_*. \qquad \text{(SA.35)}$$

Since the net transfer profile r minimizes the total promises transfer, and we already constructed a stable promises profile in Lemma SA.3 with total promises transfer $\mathcal{T}_*$, then we must have $\mathcal{T}_r = \mathcal{T}_*$, and thus all the inequalities in (SA.35) are equalities. In particular, the second inequality in (SA.35) implies that $u_* = y_*$. Moreover, by (SA.29) and (SA.30) we have $r_{\tilde{k}_*} < 0$, so that the first inequality in (SA.35) implies that $\tilde{k}_* = k_*$. Now it can be directly checked that the conditions (SA.4)-(SA.7) of Proposition SA.4 hold. This concludes the proof of the only if part of Proposition SA.4.

Step 2. In this step, we show the if part: we fix $r \in \mathcal{P}$ that satisfies conditions (SA.4)-(SA.6) of Proposition SA.4 and show that $r \in \boldsymbol{\mathcal{SM}}_R$ and that (SA.7) holds.

To show that $\boldsymbol{r} \in \mathcal{S}_R$, set $\mathcal{C} := \mathcal{C}^S \cup \underline{\mathcal{C}}^R$ in Lemma SA.1 (ii) so that $|\mathcal{C}| = \widehat{\kappa}$. Using condition (SA.6) and observing that $u_j + r_j = u_*$ for all $j \ge k_*$ (condition (SA.5)) shows that we have $u_i + r_i \le u_j + r_j$ for all $i \in \mathcal{C}$ and $j \notin \mathcal{C}$. Moreover, since $r \in \mathcal{P}$ and $r_j = -u_j + u_*$ for $j \ge \kappa_*$, we have

$$\begin{aligned} \sum_{i \in \mathcal{C}} (u_i + r_i) &= \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i - \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} r_j = \sum_{i \in \mathcal{C}^S \cup \underline{\mathcal{C}}^R} u_i - \sum_{j \in \mathcal{C}^R \setminus \underline{\mathcal{C}}^R} [-u_j + u_*] \\ &= \sum_{i \in \mathbb{I}} u_i - (\kappa - 1) u_* = 0. \end{aligned}$$

Then by applying Lemma SA.1 (ii) we see that $r \in \boldsymbol{\mathcal{S}}_R$. Moreover,

$$\mathcal{T}_r = \sum_{j \ge k_*} (-r_j) = \sum_{j \ge k_*} [u_j - u_*] = \mathcal{T}_*.$$

By Proposition 3 there exists $r^* \in \boldsymbol{\mathcal{SM}}_R$. By Step 1 (the only if direction), the net transfer profile r^* satisfies conditions (SA.4)-(SA.7), in particular, $\mathcal{T}_{r^*} = \mathcal{T}_*$, thus $\mathcal{T}_*$ is the minimum total promises transfer for all $r \in \boldsymbol{\mathcal{S}}_R$. Finally, if $r \in \boldsymbol{\mathcal{S}}_R$ satisfies (SA.4)-(SA.6), we have $\mathcal{T}_r = \mathcal{T}_*$, so r matches the minimum total promises transfer, and thus $r \in \boldsymbol{\mathcal{SM}}_R$. □

Supplemental Appendix B (SB): Reaching across the aisle transfer promises.

In this appendix, we provide necessary and sufficient conditions under which the SM equilibrium transfer promises are of the reaching across the aisle type. The following proposition generalizes the discussions from sub-section 7.1 on whether the equilibrium rules out circling the wagon transfers where some promises recipients are reform supporters. We state the proposition, discuss it and provide its proof.

Proposition SB.1. *[**Reaching across the aisle equilibria.**] When $\kappa \geq 2$, all the SM equilibrium transfer promises are of the reaching across the aisle type if and only if one of the following conditions hold:*

1. *Reform supporters lack voting power to enact the reform, $|\mathcal{C}^R| < \kappa$, as in Proposition SA.1.*
2. *Reform supporters have enough voting power, $|\mathcal{C}^R| \geq \kappa$, and $G^S \leq \Delta U_{\widehat{\kappa}}$ as in Proposition SA.3, and one of the two following conditions are satisfied*
   - *(a) $G^S = \Delta U_{\hat{\kappa}}$, and the weakest reform supporters $\underline{\mathcal{C}}^R = \{n+1, \cdots, \widehat{\kappa}\}$ have equal utilities: $u_{n+1} = \cdots = u_{\hat{\kappa}}$.*
   - *(b) $G^S < \Delta U_{\hat{\kappa}}$ and the reform supporter $\widehat{\kappa}+1$ has a utility that is equal to that of the members $\{n+1, \cdots, \widehat{\kappa}\}$: $u_{n+1} = \cdots = u_{\hat{\kappa}} = u_{\hat{\kappa}+1}$.*
3. *Reform supporters have enough voting power, $|\mathcal{C}^R| \geq \kappa$, and $\Delta U_{\widehat{\kappa}} < G^S$ as in Proposition SA.4, and $u_{n+1} = \cdots = u_{k_*-1} = u_*$.*

Proposition SB.1 shows that in the case of a majority coercion covered in Proposition SA.1, promises recipients are reform opponents in all SM equilibria. Despite their multiplicity, equilibrium transfers share the common feature of being of the reaching across the aisle type. By contrast, in the cases covered in Proposition SA.3 and Proposition SA.4 where there are enough supporters to enact the reform to begin with

($|\mathcal{C}^R| \geq \kappa$), we show in Proposition SB.1 that for the SM equilibrium transfers to always be of the reaching across the aisle type, additional restrictions are required. To rule out circle the wagon type transfers, we broadly need a stale distribution of intensities among a specific subset of reform supporters with weakest intensities. In words, the proposition shows that when the weakest reform supporters derive uniform utility from the reform, then equilibrium requires that all promises recipients are reform opponents. We now give the proof of Proposition SB.1.

Proof of Proposition SB.1: When $|\mathcal{C}^R| < \kappa$, it is clear from Proposition SA.1 that all promises recipients are reform opponents, and thus statement 1 in Proposition SB.1 holds.

We now prove statement 2 of Proposition SB.1. We first show that properties 2.a. and 2.b of Proposition SB.1 imply that any $r \in \boldsymbol{\mathcal{SM}}_R$ belongs to the reaching across the aisle type.

Consider now the first subcase where $u_{n+1} = \cdots = u_{\widehat{k}}$ and $G^S = \Delta U_{\widehat{k}}$. Note that

$$u_j + r_j = \widehat{\pi}_j(r) \geq \widehat{\pi}_{\hat{\kappa}}(r) \geq u_{\hat{\kappa}}, \quad \forall j > \hat{\kappa}.$$

Then

$$\mathcal{T}_r = \sum_{j>\hat{\kappa}} (-r_j) \leq \sum_{j>\hat{\kappa}} (u_j - u_{\hat{\kappa}}) = \Delta U_{\hat{\kappa}} = G^S.$$

Since $r \in \boldsymbol{\mathcal{SM}}_R$, then $\mathcal{T}_r = G^S$ (Proposition SA.3, condition 3.), and thus equality holds above. This implies that $-r_j = u_j - u_{\hat{\kappa}}$, and thus $\widehat{\pi}_j(r) = u_{\hat{\kappa}}$ for all $j > \hat{\kappa}$. Note further that $u_i \leq \widehat{\pi}_i(r) \leq \widehat{\pi}_j(r) = u_{\hat{\kappa}}$ for all $n < i \leq \hat{\kappa} < j$. By the assumption in this subcase, we see that $r_i = 0$ for $n < i \leq \hat{\kappa}$. Then the net transfer profile r is of the reaching across the aisle type.

Consider the second subcase where $u_{n+1} = \cdots = u_{\widehat{k}} = u_{\widehat{k}+1}$. By Proposition SA.3 Part 1, we have $r_i \geq 0$ for $n < i \leq \hat{\kappa}$ and $r_{\hat{\kappa}+1} \leq 0$. Then $\widehat{\pi}_i(r) \geq u_i = u_{\hat{\kappa}+1} \geq \widehat{\pi}_{\hat{\kappa}+1}(r)$. By Proposition SA.3 Part 2, we have $\widehat{\pi}_{\hat{\kappa}+1}(r) \geq \widehat{\pi}_i(r)$, and thus we must have

$\widehat{\pi}_{\hat{\kappa}+1}(r) = \widehat{\pi}_i(r)$. Thus, $r_i = 0$ for $n < i \leq \hat{\kappa}$, and therefore r is of the reaching across the aisle type promise.

We next prove the only if part of statement 2 in Proposition SA.1. To do so, we assume that both statement 2.a and statement 2.b are false, and construct an equilibrium promises profile $r \in \boldsymbol{\mathcal{SM}}_R$ where some promises recipients are reform supporters. Note that, when both statements 2.a and 2.b are false, the assumption that $0 < G^S \leq \Delta U_{\hat{\kappa}}$ implies that one of the following two statements must be true:

$$u_{n+1} < u_{\hat{\kappa}} \quad \text{and} \quad 0 < G^S \leq \Delta U_{\hat{\kappa}}; \tag{SB.1}$$

$$u_{n+1} = \cdots = u_{\hat{\kappa}} < u_{\hat{\kappa}+1} \quad \text{and} \quad 0 < G^S < \Delta U_{\hat{\kappa}}. \tag{SB.2}$$

Now let $0 < \varepsilon < G^S$ and we modify the equilibrium promises profile in (SA.18) as follows:

$$\begin{gathered} r_i := -\frac{G^S - \varepsilon}{U^S} u_i, \ i \in \mathcal{C}^S; \quad r_{n+1} := \varepsilon; \\ r_j := 0, \ n+1 < j \leq \hat{\kappa}; \quad r_k := -\frac{G^S}{\Delta U_{\widehat{\kappa}}}[u_k - u_{\widehat{\kappa}}], \ k \in \mathcal{C}^R \backslash \underline{\mathcal{C}}^R. \end{gathered} \tag{SB.3}$$

It can be directly checked that $r \in \mathcal{P}$ and satisfies conditions (SA.1) and (SA.3) from Proposition SA.3, so to establish that r is an SM equilibrium, we only need to prove (SA.2).

In the sub-case (SB.1), assume further that $\varepsilon < u_{\hat{\kappa}} - u_{n+1}$. Then one can check

$$\widehat{\pi}_i(r) \leq 0 \leq \widehat{\pi}_j(r) \leq u_{\hat{\kappa}} \leq \widehat{\pi}_k(r), \quad \text{for all } i \leq n < j \leq \hat{\kappa} < k.$$

In the sub-case (SB.2), assume further that $\varepsilon < [1 - \frac{G^S}{\Delta U_{\hat{\kappa}}}][u_{\hat{\kappa}+1} - u_{\hat{\kappa}}]$. Then using $u_{n+1} = u_{\widehat{\kappa}}$ one can see that, for all $i \leq n < j \leq \hat{\kappa} < k$,

$$\widehat{\pi}_i(r) \leq 0 \leq \widehat{\pi}_j(r) \leq u_{\hat{\kappa}} + \varepsilon \leq u_{\hat{\kappa}} + [1 - \frac{G^S}{\Delta U_{\hat{\kappa}}}][u_{\hat{\kappa}+1} - u_{\hat{\kappa}}] \leq \widehat{\pi}_k(r).$$

To sum up, in all subcases, the promises profile r defined in (SB.3) also satisfies the condition (SA.2) of Proposition SA.3 when ε is small enough, and as a result, $r \in \boldsymbol{\mathcal{SM}}_R$. We conclude then by observing that since $r_{n+1} > 0$, the reform supporter $n+1$ is a promise recipient, and therefore the equilibrium r has some circle the wagon transfers.

We now prove statement 3 in Proposition SB.1. We first show the if part. Fix an arbitrary $r \in \boldsymbol{\mathcal{SM}}_R$. By Proposition SA.4, we have $r_i \geq 0$ and $u_i \leq \widehat{\pi}_i(r) \leq u_*$ for $n+1 \leq i \leq \hat{\kappa}$. However, since we assume $u_{n+1} = \cdots = u_{k_*-1} = u_*$ here, we must have $r_i = 0$ for $n+1 \leq i \leq \hat{\kappa}$. Since $r_j = -(u_j - u_*) \geq 0$ for $j \geq k_*$, we see that the net transfer profile r is of the reaching across the aisle type.

We now prove the only if part. Recalling that the utilities u_i are ordered, we assume $u_{n+1} < u_*$ and we shall construct an $r \in \boldsymbol{\mathcal{SM}}_R$ which has some circle the wagon type transfers. Let $0 < \varepsilon < \mathcal{T}_*$ and we modify the promises profile described in (SA.26) as follows:

$$\begin{aligned} r_i &:= -\frac{\mathcal{T}_* - \varepsilon}{U^S} u_i,\ i \leq n; \qquad r_{n+1} := \varepsilon; \\ r_k &:= 0,\ n+1 < k < k_*; \quad r_j := -[u_k - u_*],\ j \geq k_*. \end{aligned} \tag{SB.4}$$

Assume further that $\varepsilon < u_* - u_{n+1}$, then one can check that

$$\widehat{\pi}_i(r) \leq 0 \leq \widehat{\pi}_j(r) \leq u_* = \widehat{\pi}_k(r), \quad \text{for all } i \leq n < j \leq \hat{\kappa} < k.$$

It can be checked that $r \in \mathcal{P}$ and satisfies all the other requirements in Proposition SA.4. We conclude by observing that $r_{n+1} > 0$, and therefore the SM equilibrium net transfer profile r constructed in (SB.4) has some circle the wagon transfers. $\square$