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Contract Theory in Continuous- Time Models

 Springer

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*To my parents Antun and Vjera
To my wife Ying and my son Albert*

Preface

Why We Wrote This Book In recent years there has been a significant increase in interest in continuous-time Principal–Agent models and their applications. Even though the approach is technical in nature, it often leads to elegant solutions with clear economic predictions. Our monograph sets out to survey some of the literature in a systematic way, using a general theoretical framework. The framework we find natural and general enough to include most of the existing results is the use of the so-called Stochastic Maximum Principle, in models driven by Brownian Motion. It is basically the Stochastic Calculus of Variations, used to find first order conditions for optimality. This leads to the characterization of optimal contracts through a system of Forward-Backward Stochastic Differential Equations (FBSDE’s). Even though there is no general existence theory for the FBSDE’s that appear in this context, in a number of special cases they can be solved explicitly, thus leading to the analytic form of optimal contracts, and enabling derivation of many qualitative economic conclusions. When assuming Markovian models, we can also identify sufficient conditions via the standard approach of using Hamilton–Jacobi–Bellman Partial Differential Equations (HJB PDE’s).

Who Is It For This book is aimed at researchers and graduate students in Economic Theory, Mathematical Economics and Finance, and Mathematics. It provides a general methodological framework, which, hopefully, can be used to develop further advances, both in applications and in theory. It also presents, in its last part, a primer on BSDE’s and FBSDE’s. We have used the material from the book when teaching PhD courses in contract theory at Caltech and at the University of Zagreb.

Prerequisites A solid knowledge of Stochastic Calculus and the theory of SDE’s is required, although the reader not interested in the proofs will need more of an intuitive understanding of the related mathematical concepts, than a familiarity with the technical details of the mathematical theory. A knowledge of Microeconomics is also helpful, although nothing more than a basic understanding of utility functions is required.

Structure of the Book We have divided the book into an introduction, three main middle parts, and the last part. The introduction describes the three main settings: risk sharing, hidden actions and hidden types. It also presents a simple example of each. Then, each middle part presents a general theory for the three settings, with a variety of special cases and applications. The last part presents the basics of the BSDE's theory and the FBSDE's theory.

Web Page for This Book sites.google.com/site/contracttheorycvitaniczhang/. This is a link to the book web page that will be regularly updated with material related to the book, such as corrections of typos.

Acknowledgements Our foremost gratitude goes to our families for the understanding and overall support they provided during the times we spent working on our joint research leading to this book, and for the work on the book itself. We are grateful for the support from the staff of Springer, especially Catriona Byrne, Marina Reizakis and Annika Elting. A number of colleagues and students have made useful comments and suggestions, and pointed out errors in the working manuscript, including Jin Ma, Ajay Subramanian, Xuhu Wan, Xunyu Zhou, Hualei Chang and Nikola Sandrić, and anonymous reviewers.

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Of course, we are solely responsible for any remaining errors, and the opinions, findings and conclusions or suggestions in this book do not necessarily reflect anyone's opinions but the authors'.

Final Word We hope that you will find the subject of this book interesting in its economic content, and elegant in its mathematical execution. We would be grateful to the careful reader who could inform us of any remaining typos and errors noticed, or any other comments, by sending an e-mail to our current e-mail addresses. Enjoy!

Los Angeles, USA
April 2012

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