Optimal Portfolio Composition for Sovereign Wealth Funds

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Outline

- Introductory remarks.
- Paper’s objective.
- Overview of sovereign wealth funds (SWFs).
- Modelling framework.
- Model calibration.
- Optimal allocation and consumption path.
- Concluding remarks.
Introductory remarks

- Significance of SWFs in international financial markets.
  - US$ 7.2 trillion in total assets.
  - 80% of funds (by assets) in Middle East and Asia.
  - 56% of total assets are in oil-based funds.

- Significance of SWFs for their respective economies.
  - Large assets relative to GDP and annual oil income.
  - Small ratio of assets to oil reserves (except Norway).

- The investment strategies of large oil-based SWFs:
  - Gradual increase of equity share to roughly 60%.
  - Rest is divided between bonds and alternative investments.

- Transparency and governance issues.
  - Agency problems.
  - Home/regional bias in investment strategies.
  - Fiscal rules (or lack thereof).
### SWFs Assets Relative to Output, Revenues and Oil Reserves

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Norway</td>
<td>825</td>
<td>526</td>
<td>258</td>
<td>1.73</td>
<td>20.7</td>
<td>1.56</td>
</tr>
<tr>
<td>UAE (ADIA)</td>
<td>773</td>
<td>9,584</td>
<td>4,597</td>
<td>1.94</td>
<td>8.95</td>
<td>0.08</td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>632</td>
<td>26,255</td>
<td>12,610</td>
<td>0.98</td>
<td>2.25</td>
<td>0.03</td>
</tr>
<tr>
<td>Kuwait</td>
<td>592</td>
<td>10,192</td>
<td>4,888</td>
<td>3.35</td>
<td>5.83</td>
<td>0.05</td>
</tr>
<tr>
<td>Qatar</td>
<td>256</td>
<td>2,487</td>
<td>1,187</td>
<td>0.81</td>
<td>3.46</td>
<td>0.05</td>
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<tr>
<td>Kazakhstan</td>
<td>77</td>
<td>2,940</td>
<td>1,410</td>
<td>0.35</td>
<td>1.49</td>
<td>0.02</td>
</tr>
<tr>
<td>Russia</td>
<td>74</td>
<td>7,840</td>
<td>3,760</td>
<td>0.04</td>
<td>0.31</td>
<td>0.01</td>
</tr>
<tr>
<td>Iran</td>
<td>62</td>
<td>15,149</td>
<td>7,417</td>
<td>0.15</td>
<td>0.64</td>
<td>0.00</td>
</tr>
<tr>
<td>Algeria</td>
<td>50</td>
<td>1,196</td>
<td>573</td>
<td>0.23</td>
<td>1.08</td>
<td>0.06</td>
</tr>
</tbody>
</table>
Introductory remarks: SWFs asset allocations in 2014

![Graph showing SWFs asset allocations in 2014]

- Equity
- Bonds
- Other

Countries: Norway, UAE - ADIA, Saudi Arabia, Kuwait, Qatar
Paper’s objective

Our objective is to determine the optimal asset allocation for a SWF based on income from oil. The main issues are:

- Investment horizon.
- Utility function (risk aversion and time preference).
- Nature of the available investment opportunity.
- Oil income subject to random shocks and correlated with risky asset.

The questions of interest:

- Should a risky asset (e.g. equity) be used to hedge against oil shocks?
- Should the rate of oil extraction be an additional control variable? In other words, should we solve for the optimal tradeoff between above ground and underground wealth?
- What happens if risk aversion is delinked from the rate of time preference.
- Should other state variables be included in the problem? If so, what state variables?
Our paper draws on the following strands of the literature:

- **Optimal asset allocation for a SWF.**
  - Gintschel & Scherer (2008); Scherer (2009); van den Bremer, van der Ploeg & Wills (2016).

- **Asset allocation given a stochastic stream of income.**

- **Optimal rate of extraction for an exhaustible natural resource.**
  - Hotelling (1931).
Modelling framework: notation

- $F_t$ is the value of the fund at time $t$, i.e. at the beginning of the period $[t, t+1]$.
- $Y_t$ is the income from oil allocated to the fund at time $t$:
  \[ Y_{t+1} = Y_t \exp(g + \xi_{t+1}), \quad V_t(\xi_{t+1}) = \sigma^2_\xi. \]
- $C_t$ is the consumption out of the fund over the interval $[t, t+1]$ evaluated at time $t$.

Financial Market Assumptions

- One riskless asset with continuously compounded return $r_0$.
- $(1 - \pi_t)$ is the share invested in the riskless asset.
- One risky asset with continuously compounded return $r_{1,t}$.
- $\pi_t$ is the share invested in the risky asset.
- The excess return on the risky asset obeys:
  \[ r_{1,t+1} - r_0 = \mu + u_{t+1}, \quad V_t(u_{t+1}) = \sigma^2_u. \]
- Oil income shocks ($\xi_{t+1}$) and financial shocks ($u_{t+1}$) are correlated:
  \[ \text{Cov}_t(u_{t+1}, \xi_{t+1}) = \sigma_{u\xi}. \]
The objective of the fund manager is to maximize the expected present value of future consumption at discount rate $\delta$:

$$\max_{\{C_t, \pi_t\}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \delta^t U(C_t) \right], \quad U(C_t) = \frac{C_t^{1-\gamma}}{1 - \gamma}.$$ 

subject to the intertemporal budget constraint:

$$F_{t+1} = (F_t + Y_t - C_t) R_{F,t},$$

where $R_{F,t}$ is the return on the fund:

$$R_{F,t} = \pi_t R_{1,t} + (1 - \pi_t) R_0,$$

$$R_0 = \exp(r_0),$$

$$R_{1,t} = \exp(r_{1,t}).$$
The Euler equations are given by

\[ U'(C_t) = E_t \left[ \gamma U'(C_{t+1}) R_{i,t+1} \right], \quad i = 0, 1, F. \]

To solve for the optimal allocation and the optimal path for \( C_t \), we adopt the log-linear approximation method of Campbell (1993, 1996). Write the constraint as

\[ \frac{F_{t+1}}{Y_{t+1}} = \left( \frac{F_t}{Y_t} + 1 - \frac{C_t}{Y_t} \right) \frac{Y_t}{Y_{t+1}} R_{F,t}. \]

This is equivalent in logs to

\[ f_{t+1} - y_{t+1} = \log (1 + \exp\{f_t - y_t\} - \exp\{c_t - y_t\}) - \Delta y_{t+1} + r_{F,t+1}, \]

where the lower case variables denote the log of the corresponding upper case variables.
Modelling framework: solution

Under the log-linear approximation, the optimal log consumption and portfolio composition are (lower case variables are in logs):

\[ c_t - y_t = a + b(f_t - y_t), \]  
\[ \pi_t = \mu + \frac{\sigma^2_u}{\gamma b \sigma^2_u} - \frac{1 - b \sigma_{u \xi}}{b \sigma^2_u} \]  

where

\[ a = b \frac{k + E_t[r_{F,t+1}] - g - \frac{1}{b} \lambda_t}{\rho_c}, \]

\[ b = \frac{\rho_f - 1}{\rho_c}, \]

with

\[ \lambda_t = \frac{1}{\gamma} \left[ E_t[r_{F,t+1}] + \frac{1}{2} V_t[r_{F,t+1} - \gamma(c_{t+1} - c_t)] + \log(\delta) \right] - g, \]
\[ \rho_f = \frac{\exp\{E[f_t - y_t]\}}{1 + \exp\{E[f_t - y_t]\} - \exp\{E[c_t - y_t]\}}, \]
\[ \rho_c = \frac{\exp\{E[c_t - y_t]\}}{1 + \exp\{E[f_t - y_t]\} - \exp\{E[c_t - y_t]\}}. \]
Model calibration

- We use monthly data on the oil price, S&P 500 index (as proxy for global equity) and the U.S. treasury 3-months bill rate to estimate the first and second sample moments.
- We fit models for the conditional moments: namely ARMA(2,1) for the oil price conditional mean, and TARCH(1,1,1) model (Glosten, Jagannathan and Runkle (1993)) for the conditional variance of oil and excess equity returns.
- We also fit a dynamic conditional correlations model (Engle (2002)) to estimate the conditional correlation between oil and equity.
- The purpose of the empirical models is to investigate the range of plausible values for the key parameters:
  \[ \sigma^2_u : 0.002 \]
  \[ \sigma^2_\xi : 0.006 \]
  \[ \sigma_{u\xi} \] such that \( \text{corr}(u, \xi) : [-0.7, -0.3, 0.0, 0.3, 0.7] \)
- We also check the impact of changes on \( \delta \) and \( \gamma \) on the results.
Model calibration

Risky asset excess returns conditional volatility

Oil log-price change conditional volatility

Ratio of oil-to-equity conditional volatilities
Allocation to risky asset: Negative covariance with oil

![Allocation to Risky Asset (Assuming negative covariance with oil)](image)

- $\sigma_u^2 = 0.001$
- $\sigma_u^2 = 0.002$
- $\sigma_u^2 = 0.006$
Allocation to risky asset: Positive covariance with oil

![Graph showing allocation to risky asset with positive covariance with oil over different months and for different variances.](image-url)
Wealth to income ratio for varying risk aversion

<table>
<thead>
<tr>
<th>Investment Horizon</th>
<th>$\gamma = 4$</th>
<th>$\gamma = 6$</th>
<th>$\gamma = 8$</th>
<th>$\gamma = 10$</th>
<th>$\gamma = 12$</th>
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<tbody>
<tr>
<td>2 years</td>
<td>7.40</td>
<td>7.58</td>
<td>7.67</td>
<td>7.72</td>
<td>7.76</td>
</tr>
<tr>
<td>4 years</td>
<td>6.79</td>
<td>7.15</td>
<td>7.33</td>
<td>7.44</td>
<td>7.51</td>
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<tr>
<td>6 years</td>
<td>6.16</td>
<td>6.71</td>
<td>6.99</td>
<td>7.15</td>
<td>7.26</td>
</tr>
<tr>
<td>8 years</td>
<td>5.51</td>
<td>6.27</td>
<td>6.64</td>
<td>6.86</td>
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<tr>
<td>10 years</td>
<td>4.81</td>
<td>5.81</td>
<td>6.29</td>
<td>6.57</td>
<td>6.76</td>
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</table>
Allocation to risky asset at a 10-year investment horizon

<table>
<thead>
<tr>
<th>Relative risk aversion</th>
<th>$\delta = 0.90$</th>
<th>$\delta = 0.925$</th>
<th>$\delta = 0.95$</th>
<th>$\delta = 0.975$</th>
<th>$\delta = 0.99$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 3$</td>
<td>0.20</td>
<td>0.40</td>
<td>0.47</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>$\gamma = 4$</td>
<td>0.21</td>
<td>0.30</td>
<td>0.34</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>$\gamma = 5$</td>
<td>0.18</td>
<td>0.23</td>
<td>0.25</td>
<td>0.26</td>
<td>0.26</td>
</tr>
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</table>
Concluding remarks

This is still work in progress, and we are currently extending the paper in three directions:

- Solving the model using the recursive utility function of Epstein & Zin (1989):

\[
V_t = \left\{ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left[ E_t \left( V_{t+1}^{1-\gamma} \right) \right] \right\}^{\frac{\theta}{1-\gamma}}
\]

where \( \gamma \) is the coefficient of relative risk aversion, and \( \theta > 0 \) is the elasticity of intertemporal substitution.

- Allowing for more than one risky-asset (e.g. long-term bonds), a straightforward extension given the current model structure.

- Writing \( Y_t = P_{o,t} X_t \), where \( P_{o,t} \) is the oil price and \( X_t \) is the amount of oil extracted, then using \( X_t \) as a control variable to additionally solve for the optimal rate of extraction.