1. Use Sylow’s theorems and other results to describe, up to isomorphism, the possible structures of a group of order 1005.

2. Let $R$ be a commutative ring with 1. Let $M, N$ and $V$ be $R$-modules.
   
   (a) Show if that $M$ and $N$ are projective, then so is $M \otimes_R N$.
   
   (b) Let $\text{Tr}(V) := \{\sum_i \phi_i(v_i) | \phi \in \text{Hom}_R(V,R), v_i \in V\} \subset R$. If $1 \in \text{Tr}(V)$, show that up to isomorphism $R$ is a direct summand of $V^k$ for some $k$.

3. Let $F$ be a field and $M$ a maximal ideal of $F[x_1, \ldots, x_n]$. Let $K$ be an algebraic closure of $F$. Show that $M$ is contained in at least 1 and in only finitely many maximal ideals of $K[x_1, \ldots, x_n]$.

4. Let $F$ be a finite field.
   
   (a) Show that there are irreducible polynomials over $F$ of every positive degree.
   
   (b) Show that $x^4 + 1$ is irreducible over $\mathbb{Q}[x]$ but is reducible over $\mathbb{F}_p[x]$ for every prime $p$ (hint: show there is a root in $\mathbb{F}_{p^2}$).

5. Let $F$ be a field and $M$ a finitely generated $F[x]$-module. Show that $M$ is artinian if and only if $\dim_F M$ is finite.

6. Let $R$ be a right Artinian ring with with a faithful irreducible right $R$-module. If $x, y \in R$, set $[x, y] := xy - yx$. Show that if $[[x, y], z] = 0$ for all $x, y, z \in R$, then $R$ has no nilpotent elements.