(1) Up to isomorphism, describe all groups of order 495.

(2) Let \( x^4 - 7 \in F[x] \) for \( F \subseteq \mathbb{C} \). If \( F \subseteq M \subseteq \mathbb{C} \) and \( M \) is a splitting field for \( x^4 - 7 \) over \( F \), find \( \text{Gal}(M/F) \) when
   (a) \( F = \mathbb{Q} \)
   (b) \( F = \mathbb{Q}[\sqrt{7}] \)
   (c) \( F = \mathbb{Q}[i] \), with \( i^2 = -1 \)

(3) Let \( M \) be a finitely generated \( F[x] \) module (\( F \) a field). If every submodule of \( M \) has a complement, describe the structure of \( M \) in terms of \( F[x] \). (Recall that a submodule \( H \) of a module \( M \) has a complement if there is a submodule \( H' \) so that \( M \cong H \oplus H' \), i.e. \( H + H' = M \) and \( H \cap H' = (0) \).

(4) Show that some power of \( (x + y)(x^2 + y^4 - 2) \) is in the ideal of \( \mathbb{C}[x,y] \) generated by \( x^3 + y^2 \) and \( y^3 + xy \).

(5) Let \( R \) be a commutative Noetherian ring with no nonzero nilpotent element. Set \( A = \{ \text{ann} \ I \mid I \text{ is a nonzero ideal of } R \} \) and \( M = \{ \text{maximal elements in } A \} \). Prove that \( R \) embeds in a direct sum of finitely many domains as follows:
   (a) Show that the elements of \( M \) are prime ideals in \( R \).
   (b) For \( P \neq Q \) in \( M \), show \( \text{ann} \ Q \subseteq P \).
   (c) Show that \( M \) is finite (consider sums of \( \text{ann} \ P_i \) for \( P_i \in M \)).
   (d) Show that the intersection of the elements in \( M \) is zero.

(6) Let \( R \) be a finite dimensional algebra over the field \( F \). Assume that for every \( r \in R \), there exists \( g(x) \in F[x] \), depending on \( r \), so that \( r + g(r)r^2 = 0 \). Determine the structure of \( R \).