(1) Up to isomorphism, determine all groups of order $7^2 \cdot 11^2 \cdot 19$.

(2) Let $G$ be a finite Abelian group and recall that the exponent of $G$ is the smallest positive integer $n$ so that $g^n = e_G$ for all $g \in G$. Show that $G$ is a cyclic group if and only if the order and exponent of $G$ are equal.

(3) An ideal in a commutative ring is called irreducible if it cannot be written as an intersection of finitely many properly larger ideals. If $A = \mathbb{Z}[x_1, \ldots, x_n]$ is the polynomial ring in $n$ variables over $\mathbb{Z}$, show that any ideal of $A$ is an intersection of finitely many irreducible ideals.

(4) Let $K$ be a splitting field over $\mathbb{Q}$ of the polynomial $x^{11} - 17$. Show that $\text{Gal}(K/\mathbb{Q})$ is isomorphic to the group of matrices $G = \left\{ \begin{pmatrix} 1 & a \\ 0 & b \end{pmatrix} \in \text{GL}_2(\mathbb{Z}/11\mathbb{Z}) \right\}$.

(5) Let $F$ be a filed and $M$ an irreducible (i.e. simple and nontrivial) $F[x_1, \ldots, x_n]$ module.
   (a) If $F$ is algebraically closed, show that $\text{dim}_F M = 1$.
   (b) For any $F$ show that $\text{dim}_F M$ is finite.

(6) Let $R$ be a finite ring in which every element is a sum of nilpotent elements. Show that $R$ is nilpotent. (Hint: What is the traces of a nilpotent element in $M_n(F)$ for $F$ a field?)