1) Find up to isomorphism all groups of order $3 \cdot 7 \cdot 19 \cdot 37$.

2) Let $D$ be a commutative domain with multiplicative identity 1 and assume that the additive group $D$ is finitely generated. Prove
   (a) Characteristic $K = 0$ if and only if $(D, +)$ is a free abelian group.
   (b) If $\exists$ an integer $n > 1$ such that $f : D \to D$ defined by $x \mapsto nx$ is onto then $D$ is a finite field.
   (c) If $M$ is a maximal ideal in $D$ then $M \cap i(\mathbb{Z}) = \pi(\mathbb{Z})$ for some prime $p$, $i(\mathbb{Z}) = \{n \cdot 1, n \in \mathbb{Z}\}$.

3) Let $f(x) \in \mathbb{Q}(x)$ be irreducible with $\deg f = n$. Let $M \subset \mathbb{C}$ be a splitting field for $f(x)$ over $\mathbb{Q}$.
   (a) Show that if $\text{Gal}(M/\mathbb{Q})$ is abelian then every subfield field of $M$ is Galois over $\mathbb{Q}$.
   (b) Show that if $\text{Gal}(M/\mathbb{Q})$ is abelian then $[M : \mathbb{Q}] = n$.
   (c) Show that $F_{p^n}$ is a splitting field of $f(x)$ over $F_p$.

4) Let $f_i(x, y) = a_i x^2 + b_i x y + c_i y^2 \in \mathbb{C}[x, y], 1 \leq i \leq n$. Show that there exists $(u, v) \in \mathbb{C}^2$ such that $u^2 + v^2 = 1$, but $f_i(u, v) \neq 0 \forall i = 1, \ldots, n$.

5) Given the linear equation $a_1 x_1 + \ldots + a_t x_t = 0$, $a_i \in A = k[x_1, \ldots, x_m]$ and $k$ a field, prove that there are solutions $Y_1, \ldots, Y_q \in A^t$ such that for each solution $Y$, there exists $b_i, \ldots, b_q \in A$ such that $Y = \sum_{i=1}^q b_i Y_i$. If $A = \mathbb{Z}$, prove that you can take $q = t - 1$.

6) Let $x$ denote a fixed non zero vector in $\mathbb{C}^3$ and $A_x$ denote the ring of matrices $T \in M_3(\mathbb{C})$ such that $xT = 0$.
   (a) Prove that $A_{xU} \cong A_x$ for any $U \in \text{GL}_3(\mathbb{C})$, hence $A_x \cong A_y$ for any non zero $y \in \mathbb{C}^3$.
   (b) Prove that $\{(a_{ij}) \in A_{(1,0,0)} : a_{ij} = 0 \text{ for } j > 1\}$ is nilpotent ideal in $A_{(1,0,0)}$.
   (c) Prove that the Jacobson radical $J(A_x)$ is not zero and that $A_x/J(A_x) \cong M_2(\mathbb{C})$. 