
Using Russell’s Theory of Descriptions to Solve Logical Puzzles

One of Russell’s puzzles involves the law of classical logic called the law of the excluded middle. Here is his statement of the puzzle, which involves sentences (37a-c).

“By the law of excluded middle, either “A is B” or “A is not B” must be true. Hence either “The present King of France is bald” or “The present King of France is not bald” must be true. Yet if we enumerated the things that are bald and the things that are not bald, we should not find the present King of France in either list. Hegelians, who love a synthesis, will probably conclude that he wears a wig.” (485)

1a. The present King of France is bald.
1b. The present King of France is not bald.
1c. Either the present King of France is bald or the present King of France is not bald.

In the passage, Russell gives a reason for supposing that neither (1a) nor (1b) is true, which in turn seems to suggest that (1c) isn’t true. But that violates a law of classical logic that tells us that for every sentence S, [Either S or ~S] is true. Since Russell regarded the law as correct, he needed a way of defusing this apparent counterexample.

The key to doing this lies in his general rule R for determining the logical form of sentences containing definite descriptions.

R. \[ C \{the F\} \Rightarrow \exists x \forall y \{ (Fy \leftrightarrow y = x ) \& Cx \}\]

This rule says that if a definite description occurs in a sentence along with additional material C, then it can be eliminated (bringing us closer to the logical form of the sentence) by replacing the description with a variable, and introducing quantifiers plus the uniqueness clause as indicated. When we do this for (1a), C corresponds to ‘is bald’. Putting (1a) in the form of the left hand side of R, we have (1a’).

1a’. B [the present King of France]

Applying R to this gives the logical form (lfa).

lfa. \[ \exists x \forall y \{ (Kyf \leftrightarrow y = x ) \& Bx \}\]

Next, we apply R to (1b), which he would first express in the more convenient form (1b’).

1b’. ~ B [the present King of France]

There are now two ways of applying R. If we take C in (1b’) to be what it was in (1a’), namely ‘B’, then we are viewing (1b’) as (1b’1) and applying R inside the parentheses.

1b’1. ~ (B [the present King of France] )

This gives us lfb1, which is the negation of lfa.

lfb1. \[ \exists x \forall y \{ (Kyf \leftrightarrow y = x ) \& ~Bx \} \]

If we take ‘~B’ in (1b’) to play the role of C, then we treat (1b’) as (1b’2); applying R gives us (lfb2).

1b’2. ( ~ B [the present King of France] )

Thus, Russell’s rule R for relating English sentences to their logical forms yields the conclusion that negative sentences containing descriptions are ambiguous, which, in fact, they are.

The first, lfb1, may be paraphrased: It is not the case that there is someone who is both bald and unique in being King of France; the second, lfb2: There is someone who is not bald, who is unique in being King of France. The latter logically entails that there is a King of France, whereas the former does not.
Facts to notice: (lfa) is incompatible with both of the logical forms of (1b) – i.e. (lfa) and (lfb1) can’t be jointly true, and (lfa) and (lfb2) can’t be jointly true. But (lfa) and (lfb2) can be jointly untrue--when there is no unique King of France. This doesn’t violate the law of the excluded middle because (lfb2) isn’t the logical negation of (lfa). Rather, (lfb1) is the logical negation of (lfa); these two cannot both fail to be true. Thus their disjunction must be true, exactly as the law maintains. Hence the puzzle is solved.

Terminology. When (1b) is analyzed as (1b’1) and ultimately as having the logical form (lfb1), the description the present King of France is said to take narrow scope (relative to the negation operator), and to have secondary occurrence, in the sentence or proposition as a whole. When (1b) is analyzed as (1b’2), and ultimately as having the logical form (lfb2), the description is said to take wide scope over the negation operator, and to have primary occurrence in the sentence or proposition.

Another Example
2. John believes that the person sitting over there is famous.
2’. John believes that F [the person sitting over there]

Russell’s theory predicts that there are two interpretations of (2). On one interpretation, the description ‘the person sitting over there’ has narrow scope (relative to ‘believe’), and secondary occurrence in the sentence or proposition. On this interpretation, R is applied within the subordinate clause by itself to give the logical form

If1. John believes that (∃x ∀y [ (Sitting over there (y) ↔ y = x ) & Fx])

When interpreted this way, (2) tells us that John believes that there is just one person sitting over there and whoever that person may be, that person is famous. On this reading, (2) may be true, even if no one is sitting over there; it may also be true if Mary is sitting there, but John doesn’t know that she is, or think that she is famous – all that is required is that he believe that someone famous is sitting over there.

On the other interpretation of (2), the description ‘the person sitting over there’ has wide scope over the belief predicate, and primary occurrence in the sentence or proposition as a whole. On this reading, R is applied to the whole sentence to give us the logical form If2.

If2  ∃x ∀y [ (Sitting over there (y) ↔ y = x ) & John believes that Fx ].

When (2) is interpreted in this way, it tells us that there is one and only one person sitting over there and John believes that person to be famous. In order for this to be true, there really must be just one person sitting over there and John must believe that person to be famous; however it is not necessary that John have any idea where that person is, or believe that anyone is sitting over there.

George IV and The Author of Waverley
Another logical puzzle to which Russell took his theory of descriptions to provide a solution involves the interaction of a law of logic known as the substitutivity of identity with propositional attitude ascriptions (sentences with verbs like ‘believe’, ‘know’, ‘assert’, ‘doubt’, and ‘wonder’), which report a relation between an agent and a proposition.

If a is identical with b, whatever is true of the one is true of the other, and either may be substituted for the other in any proposition without altering the truth or falsehood of that proposition. Now George IV wished to know whether Scott was the author of Waverley; and in fact Scott was the author of Waverley. Hence we may substitute Scott for the author of “Waverley,” and thereby prove that George IV wished to know whether Scott was Scott. Yet an interest in the law of identity can hardly be attributed to the first gentleman of Europe. (485)

Far from a model of clarity, the passage swings inconsistently back and forth between talking about expressions and talking about the individuals that the expressions designate, as well as between talking about sentences and talking about the propositions those sentences express. Still the puzzle is clear enough. P1 and P2 appear to be true, even though the conclusion, C, appears to be false.
P1. George IV wondered whether Scott was the author of *Waverley*.

P2. Scott was the author of *Waverley* – i.e. Scott = the author of *Waverley*

C. George IV wondered whether Scott was Scott

The puzzle arises from the law of the substitutivity of identity, which may be stated as follows:

SI. When α and β are singular referring expressions, and the sentence \[α = β\] is true, α and β refer to the same thing, and so substitution of one for the other in any true sentence will always yield a true sentence.

If P1 and P2 are true, SI is truth preserving, and C follows from P1 and P2 by SI, then C must also be true. The problem is that it seems not to be. Russell claims that his theory solves the problem.

The puzzle about George IV’s curiosity is now seen to have a very simple solution. The proposition “Scott was the author of *Waverley,*” which ...[when written out in unabbreviated form], does not contain any constituent “the author of *Waverley*” for which we could substitute “Scott”. This does not interfere with the truth of inferences resulting from making what is verbally the substitution of “Scott” for “the author of *Waverley,*” so long as “the author of *Waverley*” has what I call a primary occurrence in the proposition considered. (488-89)

The second sentence of the passage contains the key idea. Since ‘the author of *Waverley*’ is a singular definite description, it is not, logically, a singular referring expression, and so it does not figure in applications of SI. (In discussing this example, Russell treats ‘Scott’ as a logically proper name.) This rule, like every logical rule, applies only to logical forms of sentences. So, to evaluate the argument, P1 and P2 have to be replaced with their logical forms.

Since P1 is a compound sentence containing a definite description, it has two readings -- one in which the description has primary occurrence, and one reading in which the description has secondary occurrence. Thus, there are two reconstructions of the argument – one corresponding to each reading.

**Argument 1: Primary Occurrence of the Description in P1**

P1:p. \[∃x \forall y \ [(y \text{ Wrote } \iff y = x) \& \text{ George IV wondered whether } x = \text{ Scott}]\]

There was exactly one person who wrote *Waverley* and George wondered whether he was Scott.

P2. \[∃x \forall y \ [(y \text{ Wrote } \iff y = x) \& x = \text{ Scott}]\]

There was one and only one person who wrote *Waverley* and he was Scott.

C. George IV wondered whether Scott = Scott.

**Argument 2: Secondary Occurrence of the Description in P1**

P1:s. George IV wondered whether \[∃x \forall y \ [(y \text{ Wrote } \iff y = x) \& x = \text{ Scott}]\]

George IV wondered whether (there was one and only one person who wrote *Waverley* and he was Scott)

P2. \[∃x \forall y \ [(y \text{ Wrote } \iff y = x) \& x = \text{ Scott}]\]

There was one and only one person who wrote *Waverley* and he was Scott.

C. George IV wondered whether Scott = Scott.

Russell takes the reading on which the description has secondary occurrence as the most natural one. With this in mind, we evaluate Argument 2. Suppose that P1s and P2 are true. Then George wondered whether a certain proposition – the one expressed by P2 – was true. And in fact, it was true. However, since P2 is not a simple identity statement \[α = β\], and there is no singular referring expression in P1s to be substituted for ‘Scott’, we cannot use the rule SI to derive C from P1s and P2. So far so good.

But this is not the whole story. In the last sentence of the passage Russell says:
“This does not interfere with the truth of inferences resulting from making what is verbally the substitution of ‘Scott’ for the author of Waverley’, so long as the author of Waverley has what I call a primary occurrence in the proposition considered.”

His point is that when the description is interpreted as having primary occurrence in P1p, the truth of the premises P1p and P2 does guarantee the truth of the conclusion C. But, according to the theory of descriptions, SI no more applies in Argument 1 than it did in Argument 2. Hence one cannot explain the difference between the two arguments, and the invalidity of Argument 2, by noting that the theory of descriptions does not allow one to apply SI by substituting ‘Scott’ for ‘the author of Waverley’.

To understand what is responsible for the difference in these arguments, it is best to begin by verifying that the step from premises to conclusion in Argument 1 is truth preserving. If P1p is true, then there is one and only one individual who wrote Waverley, and the sentence (i) ‘George IV wondered whether x = Scott’ – is true when ‘x’ is taken as a logically proper name of that individual. This in turn will be true just in case George IV wondered whether a certain proposition p – namely, the one expressed by the sentence (ii) ‘x = Scott’ – was true, again taking ‘x’ to be a logically proper name of the unique individual who wrote Waverley. If P2 is true, then this individual is Scott, in which case ‘x’ and ‘Scott’ are logically proper names of the same person. Because of this, their having the same reference guarantees that they mean the same thing, and hence that substitution of one for the other in any sentence doesn’t change the proposition expressed. From this we conclude that ‘Scott = Scott’ expresses the same proposition p as (ii) ‘x = Scott’. Since we have already established that George IV wondered whether p was true, it follows that c is also true. Thus the inference from P1p and P2 to C is guaranteed to be truth preserving, in Argument 1.

Next consider Argument 2. If P1, is true, then George IV wondered whether a certain proposition q was true – where q is the proposition that a single person wrote Waverley and that person was Scott. If P2 is true then q is, in fact, true. However, this tells us nothing about whether George IV wondered whether the proposition p, that Scott is Scott, was true. Since it is clearly possible to wonder whether q is true without wondering whether p is true, it is possible for the premises of Argument 2 to be true, while the conclusion is false. Hence the argument is invalid.

This solution supports Russell’s theory of descriptions provided there really are two ways of taking P1, both of which are capable of being true, and both of which figure in arguments, along with P2, to conclusion C, one of which is a valid argument and one of which isn’t. If this is a correct description of how we understand the English sentences P1, P2, and C, then Russell’s theory provides an elegant explanation of that fact. Does P1 really have a sense in which it could both be true and figure in such a valid argument? Russell seems to have thought so, and to have recognized the kind of situation in which P1 might be used with this reading.

“When we say ‘George IV wished to know whether Scott was the author of Waverley’, we normally mean ‘George IV wished to know whether one and only one man wrote Waverley and Scott was that man’, but we may also mean: ‘One and only one man wrote Waverley, and George IV wished to know whether Scott was that man’. In the latter, ‘the author of Waverley’ has a primary occurrence; in the former, a secondary. The latter might be expressed by ‘George IV wished to know, concerning the man who in fact wrote Waverley, whether he was Scott’. This would be true, for example, if George IV had seen Scott at a distance, and had asked ‘Is that Scott?’ (489)

A similar example, not employing a proper name for Scott, is provided by (3).

3. Mary wondered whether the author of Waverley wrote Waverley.

The most natural interpretation of (3) is one in which the description has primary occurrence – one and only one person x wrote Waverley and Mary wondered, about that person x, whether x wrote Waverley. A speaker who assertively uttered (3), intending this interpretation, might know precisely who the author
of Waverley was, and have overhead Mary say “Did he write Waverley?”, asking about the man in question. It seems clear that if (3) were used in this way, it would express a truth. It is a virtue of Russell’s theory that it explains this.

There is, however, a discordant note in Russell’s account. It occurs in the words he used when setting up the puzzle to emphasize the falsity of the conclusion C that might – but for his theory of descriptions – wrongly be derived from P1 and P2. There, Russell dismisses C with the humorous remark “Yet an interest in the law of identity can hardly be attributed to the first gentleman of Europe” – which seems to suggest that it would be absurd to suppose that anyone might doubt whether the proposition that Scott was Scott was true. What we have just seen is that his own discussion in “On Denoting” just a few pages later takes such doubt to be quite possible, and that without it his chosen example would fail to provide the support for his theory of descriptions that he (rightly) takes it to provide.

For Russell in 1905, propositions are the meanings of sentences, and the contributions made by logically proper names to propositions expressed by sentences containing them are the entities they name. On this view, if \( \alpha \) and \( \beta \) are different logically proper names of the same object, then \([\alpha = \alpha]\) and \([\alpha = \beta]\) express the same proposition. Can someone understand both sentences without knowing this, and even while accepting the former sentence and rejecting the latter? One of the passages from “On Denoting” implicitly answers the question.

“The latter [interpretation of P1, in which the description has primary occurrence] might be expressed by ‘George IV wished to know, concerning the man who in fact wrote Waverley, whether he was Scott’. This would be true, for example, if George IV had seen Scott at a distance, and had asked ‘Is that Scott?’”

Here, Russell imagines the demonstrative ‘that’ being used as a logically proper name for Scott. Since he uses ‘Scott’ as such a name throughout the discussion, the question he imagines George IV asking must be one about the proposition expressed by ‘That is Scott’, in which both singular terms are logically proper names of the same individual, and ‘is’ stands for identity. Here, Russell seems to be presupposing that when \( \alpha \) and \( \beta \) are different logically proper names of the same individual, it is possible for one to understand \([\alpha = \alpha]\) and \([\alpha = \beta]\) without knowing that they express the same proposition \( p \). In such a case, one may accept the former sentence while wondering about the truth of the latter. It is a short step from here to the view that such an agent simultaneously believes \( p \) to be true while also wondering whether \( p \) is true, because the agent doesn’t know that the proposition he or she wonders about is one that he or she already believes – and even knows.

Although this position is coherent and defensible, there is reason to believe that it wasn’t Russell’s view. We know that Russell had already decided that we are not acquainted with other minds (people), from which it followed that they couldn’t be referents of logically proper names, or constituents of propositions we can entertain. So, the use of ‘Scott’ as a logical proper name in “On Denoting” must be a useful fiction to avoid epistemological complications that might otherwise distract from the logic-linguistic theory presented there.

Moreover, at the end of “On Denoting” he indicates that matter “in the sense in which matter occurs in physics” is not among the objects of our acquaintance. By then he was on a fast track to eliminating everything physical, as well as everything mental outside oneself, from the charmed circle of possible objects of acquaintance. By 1910, he would hold that the only objects that can be referents of logically proper names are those the existence and identity of which one can’t be mistaken about. Since material objects and other human beings don’t satisfy this condition, he held that they couldn’t be the referents of logically proper names, or occur as constituents of propositions we can entertain. What he seemed not to realize is that in excluding both from our sphere of cognitive acquaintance he was losing powerful evidence for his theory of descriptions, as illustrated by his discussion of George IV.
Negative Existentials

The final logical puzzle mentioned by Russell in “On Denoting” to which his theory offers a solution is the problem of negative existentials, which he states as follows.

Consider the proposition "A differs from B". If this is true, there is a difference between A and B, which fact may be expressed in the form "the difference between A and B subsists". But if it is false that A differs from B, then there is no difference between A and B, which fact may be expressed in the form "the difference between A and B does not subsist". **But how can a non-entity be the subject of a proposition? "I think, therefore I am" is no more evident than "I am the subject of a proposition, therefore I am," provided "I am" is taken to assert subsistence or being, not existence. Hence, it would appear, it must always be self-contradictory to deny the being of anything; but we have seen, in connexion with Meinong, that to admit being also sometimes leads to contradictions. Thus if A and B do not differ, to suppose either that there is, or that there is not, such an object as "the difference between A and B" seems equally impossible.** (485)

His solution is stated as follows:

We can now see also how to deny that there is such an object as the difference between A and B in the case when A and B do not differ. If A and B do differ, there is one and only one entity x such that "x is the difference between A and B" is a true proposition; if A and B do not differ, there is no such entity x. Thus … "the difference between A and B" has a denotation when A and B differ but not otherwise. (490)

The problem making sense of this passage is figuring out how to understand the seemingly straightforward result (4) of applying Russell’s theory of definite descriptions to (4).

4a. The difference between a and b is F.
4b. ∃x ∀y [(Dyab) ↔ y = x) & Fx]

What are the entities that are supposed to count as differences between two things that are not identical? Is it true that whenever two things are different, i.e. non-identical, there is exactly one thing that is a difference between them in the relevant sense – as (4b) seems to require? Often we think that different things can differ in many respects. If so, none of the respects in which the things differ can be the difference we are looking for.

I suspect that what Russell has in mind by “the difference between A and B,” is that fact that A and B differ. On this view, what (4b) says is that there is one and only one thing x of which (i) and (ii) both hold – (i) the constituents of x are simply A, B, and the difference relation and (ii) in x A and B are (really) related by difference – and, in addition, x is F. Since I am not fond of facts, I am not fond of this ontologically inflationary way of construing (4a), but Russell seems to have been, as suggested by the way in which he completes the above passage.

This difference applies to true and false propositions generally. If "a R b" stands for "a has the relation R to b," then when a R b is true, there is such an entity as the relation R between a and b; when a R b is false, there is no such entity. Thus out of any proposition we can make a denoting phrase, which denotes an entity if the proposition is true, but does not denote an entity if the proposition is false. E.g., it is true…that the earth revolves round the sun, and false that the sun revolves round the earth; hence "the revolution of the earth round the sun" denotes an entity, while "the revolution of the sun round the earth" does not denote an entity. (490-91)

I think that what Russell has in mind is a unique fact corresponding to each true sentence. On this view, when A differs from B, there will be a fact, denoted by “the difference between A and B” of which the formula ‘∀y (Dyab) ↔ y = x’ is uniquely true, thereby guaranteeing the truth of (5).

5. There is such a thing as the difference between a and b.
   ∃x ∀y (Dyab ↔ y = x)

When A doesn’t differ from B – i.e. when they are identical – (5) will be false, and so the negative existential, (6), will be true.
6. There is no such a thing as the difference between a and b. (The difference between a and b doesn’t exist.)

\[ \sim \exists x \forall y (Dyab \leftrightarrow y = x) \]

Whatever one thinks of Russell’s free and easy way with differences and facts, the theory of descriptions isn’t to blame. It functions here to avoid the usual problems about negative existentials, just as it does in other cases.

**Problems, Challenges, and Refinements**

*The Clash between Russell’s Epistemology and his Theory of Descriptions*

When examining Russell’s solution of the puzzle about George IV, and his view of ordinary proper names (and demonstratives) vs. logically proper names, we noted the pressures leading him to restrict his circle of acquaintance, culminating in the extreme position enunciated in “Knowledge by Acquaintance and Knowledge by Description.”

To sum up our whole discussion: We began by distinguishing two sorts of knowledge of objects, namely, knowledge by *acquaintance* and knowledge by *description*. Of these it is only the former that brings the object itself before the mind. We have acquaintance with sense-data, with many universals, and possibly with ourselves, but not with physical objects or other minds. We have *descriptive* knowledge of an object when we know that it is *the* object having some property or properties with which we are acquainted; that is to say, when we know that the property or properties in question belong to one object, and no more, we are said to have knowledge of that one object by description, whether or not we are acquainted with the object. Our knowledge of physical objects and of other minds is only knowledge by description, the descriptions involved being usually such as involve sense-data. All propositions intelligible to us, whether or not they primarily concern things only known to us by description, are composed wholly of constituents with which we are acquainted, for a constituent with which we are not acquainted is unintelligible to us. (127-28)

On this view, whenever we think or talk about material objects or other people, our words describe, rather than name, them. In addition, the propositions we entertain never contain material objects or other people as constituents, but are always entirely made up of descriptive properties and relations, plus abstract logical concepts and momentary aspects of ourselves with which we are cognitively acquainted. It follows that the only singular propositions we can believe are about these things.

This internalist conception of the objects of thought has disastrous consequences for the application of his theory of descriptions to propositional attitude ascriptions. The relevant cases are examples of the type (7a) and (7b), in which \( v \) is a verb (like believe, doubt, assert, or wonder) that relates an agent to a proposition, and the complement sentence \( S \) contains a description \([\text{the } D]\) that can be interpreted as having a primary occurrence in the sentence as a whole.

7a. A v’s that \( S(\text{the } D) \)

7b. A v’s whether \( S(\text{the } D) \)

With this, the theory of descriptions says (7a) is true iff (7a\(_p\)) is true, and (7b) is true iff (7b\(_p\)) is true.

\[ 7a_p: \exists x \forall y [ (Dy \leftrightarrow y = x) \& A \text{ v’s that } S(x)] \]

\[ 7b_p: \exists x \forall y [ (Dy \leftrightarrow y = x) \& A \text{ v’s whether } S(x)] \]

These can be true only if the agent bears the relation (belief, assertion, wondering-about-the-truth-of) expressed by the verb \( v \) to the proposition expressed by \( S(x) \), when ‘\( x \)’ is a logically proper name of the unique object satisfying D. The key epistemological doctrine of the article is that agents can’t bear any such cognitive relation to that proposition when the referent of ‘\( x \)’ is a physical object or another person. So all examples of the form (7a\(_p\)),(7b\(_p\)) involving such objects are false. Russell’s radical epistemological doctrine undermines one of the most impressive applications of his theory of descriptions.
His reasoning can be partially reconstructed as follows: First, he defines a logically proper name as a term the meaning of which is its referent. Next, he assumes that one can't be mistaken about whether or not one means something by one's words. Often when I use an expression it seems I can be certain that I mean something by it, even if I am not certain that what I mean is the same as what others mean by it, and even if I am not certain that the expression really refers to anything, when it is used with the meaning I attach to it. In these cases, Russell thinks, I can be certain that I mean something by a term, even if I am not certain that the term succeeds in designating anything in the world. For example, I can be sure that I mean something when I use the words ‘the house I once owned in Princeton’ even though I am not sure that this expression refers to anything, since my former house may have burned down since I left. Similarly, Russell would say I can be sure that I mean something when I use ‘Thales’, even if I am not completely sure that there really was such a man. So, he thinks, I can know that the term ‘Thales’, as used by me, has a meaning, even if I don’t know that it has a referent.

Suppose I do use N as a logically proper name – i.e. as a term the meaning of which is its referent. Then, Russell would argue, whenever I sincerely use N to mean something, it is guaranteed to do so. Moreover, what it both means and refers to is what I take it to mean and refer to. So, the only objects that can be referents of logically proper names are objects the existence of which I couldn’t be mistaken about in any situation in which I can reasonably take myself to name them. Since material objects and other human beings don’t satisfy this condition, they can’t be referents of logically proper names. The only concrete objects that do satisfy the condition, and so can be referents of logically proper names, are oneself, one’s own thoughts, and one’s momentary sense data. So, Russell came to believe, whenever we think or talk about material objects or other people, our words describe them, rather than naming them directly. In addition, the propositions we believe never contain material objects or other people as constituents, but rather are always made up of descriptive properties and relations, plus abstract logical concepts, plus, perhaps, one or more of the ego-centric particulars with which we are acquainted.

The view is a disaster. The picture he paints is one in which agents are trapped in solipsistic worlds of Descartes’ first mediation. Their only cognitive contact with things other than themselves or their own private thoughts and experiences, is their contact with Platonic properties and relations they somehow comprehend and know to be instantiated, despite the fact that these properties and relations apply to entities they never perceive. There are three questionable philosophical views that led Russell to this incredible position: (i) his view that the objects of perception are mind-dependent sense data, and the contents of perceptual experiences are propositions about sense data, (ii) his view that cognitive and perceptual contents are transparent in the sense that one can always tell when contents A and B are the same or different, and (iii) his individualistic conception of the meaning of expressions of one’s language as dependent on the thoughts one uses its sentences to express – as opposed to a social conception in which words in a common language acquire contents that make propositions available as belief objects to agents whose only cognitive contact with constituents of those propositions is through language. The common thread running through these views is internalism – the view that the contents of all cognitive and perceptual states are entirely dependent on the internal states of the agent. In linking his insightful philosophy of language with a thoroughly internalist philosophy of mind, Russell created serious problems for his general philosophy.

No Meaning in Isolation?

Recall the first two sentences of “On Denoting.”

“By a “denoting phrase” I mean a phrase such as any one of the following: [here the reader should supply quotes around each phrase] a man, some man, any man, every man, all men, the present King of England, the present King of France, the centre of mass of the Solar System at the first instant of the twentieth century, the revolution of the earth round the sun, the revolution of the sun round the earth. Thus a phrase is doting solely in virtue of its form.”
What form did he mean? At first blush it might appear to be the form an/some/any/every/all/the F. But surely, ‘a’, ‘some’, ‘any’, ‘every’, ‘all’ and ‘the’ are merely examples of expressions that can be combined with a simple common noun, or a complex phrase functioning as such. Alongside [an F], [some F], [any F], [every F], [all Fs], and [the F], we have [each F], [exactly n Fs], [at least n Fs], [at most n Fs], [several Fs], [many F’s], [more than 20 but fewer than 50 F’s], [most Fs], [few Fs], [no Fs], and many more. These form a natural syntactic and semantic class. All are quantifiers consisting of a simple or complex common noun determining the range of quantification, together with a word or phrase (called ‘a determiner’ by linguists) determining the extent of the quantification over that range. Being quantifiers, they are complex members of the same class as the simple quantifiers ‘everything’ and ‘something’, which Russell analyzes as expressing properties of propositional functions. One of the great insights of “On Denoting” is the recognition that definite descriptions [the F] can be given a natural analysis as members of this class, instead of being classified, along with proper names and demonstratives, as singular referring expressions.

Unfortunately, Russell combines this insight with the more questionable idea that quantifiers [Det F], consisting of a determiner and a common noun, are “incomplete symbols” that “have no meaning in isolation.” Although definite descriptions were his favorite examples, every complex quantifier he discusses is treated as an “incomplete symbol.” Whenever such a quantifier Q is a grammatical constituent of a sentence S, the proposition Russell takes S to express does not contain a single constituent that corresponds to, or is the meaning of, Q. It is striking that he gives this broad incomplete-symbol claim almost no defense. The inconclusive Gray’s-Elegy argument does try to refute analyses of definite descriptions as singular referring expressions the meanings of which occur as constituents of propositions. But even if this argument were successful, it would not rule out the possibility that definite descriptions are complete symbols by virtue of being complex quantifiers. Russell shows no awareness that such quantifiers can be analyzed as contributing their meanings, as constituents, to the propositions expressed by sentences containing them.

The analysis is indicated by (8) and (9).

8a. Every F is G
\[ \forall x: Fx \] Gx
The propositional function \( p_G \) that assigns to any object \( o \) the proposition expressed by ‘Gx’ relative to an assignment of \( o \) to ‘x’ is \textit{true whenever o is F} – (i.e. \( p_G \) assigns a \textit{true} proposition to every object to which \( p_F \) assigns a true proposition).

b. Some F is G
\[ \exists x: Fx \] Gx
\( P_G \) is \textit{sometimes true when o is F} (\( p_G \) assigns a true proposition to some object to which \( p_F \) assigns a true proposition).

c. The F is G
\[ \text{The } x: Fx \] Gx
\( P_G \) is \textit{true of o when o is unique in being F} (\( p_G \) assigns a true proposition to an object that is unique in being one to which \( p_F \) assigns a true proposition).

d. Exactly n F’s are G
\[ \text{Exactly n } x: Fx \] Gx
\( P_G \) is \textit{true in exactly n cases in which o is F} (\( p_G \) assigns a true proposition to exactly n distinct objects to which \( p_F \) assigns a true proposition).
e. At least \( n \) F’s are G
\[
\text{[At least } n \text{ } x: Fx] \ Gx
\]
\( P_G \) is true in \( n \) or more cases in which \( o \) is F (\( p_G \) assigns a truth to at least \( n \) objects to which \( p_F \) assigns a truth).

9a. Most F’s are G
\[
\text{[Most } x: Fx] \ Gx
\]
\( P_G \) is true of most of the objects of which F is true (\( p_G \) assigns a truth to most of the objects that \( p_F \) assigns a truth).

b. Many F’s are G
\[
\text{[Many } x: Fx] \ Gx
\]
\( P_G \) is true is true of many of the objects of which F is true (\( p_G \) assigns a truth to many of the objects to which \( p_F \) assigns a truth).

c. Few F’s are G
\[
\text{[Few } x: Fx] \ Gx
\]
\( P_G \) is true for some, but not many, of the objects of which F is true (\( p_G \) assigns a truth to some, but not many, of the objects to which \( p_F \) assigns a truth).

It is clear that our understanding of these denoting phrases conforms to a common pattern. So, if one is describing English, rather than regimenting its sentences in a system designed for other purposes, one should treat \([\text{Det } F]\) as a semantic, as well as a syntactic, unit.

On this analysis, the denoting phrases in (10) are generalized quantifiers, and the sentences in which they occur have logical forms along the lines indicated.

10a. Every politician is slippery.
\[
\forall x: \text{Politician } x \ (\text{Slippery } x)
\]

b. Some logician is brilliant.
\[
\exists x: \text{Logician } x \ (\text{Brilliant } x)
\]

c. The man in an iron mask is innocent.
\[
\text{The } x: \text{Man in an iron mask } x \ (\text{Innocent } x)
\]

d. Several students are curious.
\[
\text{Several } x: \text{Students } x \ (\text{Curious } x)
\]

e. Most/many/few students are curious.
\[
\text{Most/many/few } x: \text{Student } x \ (\text{Curious } x)
\]

In each case, the logical form \([\text{[Det } x: Fx] \ (Gx)]\) indicates that the sentence is true iff \( p_G \) has the property expressed by the quantifier. So, (10a) is true iff the propositional function that assigns to \( o \) the proposition that \( o \) is slippery is true whenever \( o \) is a politician; (10b) is true iff the propositional function that assigns to \( o \) the proposition that \( o \) is brilliant is true in some case in which \( o \) is a logician; (10c) is true iff the propositional function that assigns to \( o \) the proposition that \( o \) is innocent is true of an object that is unique in being a man in an iron mask; (10d) is true iff the propositional function that assigns to \( o \) the proposition that \( o \) is curious is true of several students; and (10e) is true iff that propositional function is true of most, many, or some but not many, of those students.

This analysis of the truth conditions of (10e) is not just natural, but required – since one can’t get the correct truth conditions by treating ‘most’, ‘many’, or ‘few’ as simple unrestricted quantifiers (expressing the property of being true of most, many, or few things in general), and prefixing them to ‘(Student x \( \rightarrow \) Curious x)’, to ‘(Student x & Curious x)’, or to any other formula of Russell’s language.
of logical form. Given that we should provide a unified analysis of all expressions [Det F], the fact that we must treat [Most/many/few F] as generalized quantifiers (and hence as semantic units), requires anyone constructing a semantic theory of English to do the same for the other quantifier phrases as well. This is a significant modification of Russell’s theory.

Nevertheless, Russell shouldn’t be judged too harshly for missing it. He wasn’t trying to construct a semantic theory for a natural language containing infinitely many generalized quantifiers understood by speakers in a uniform way. His main concern in 1905 was with logic, mathematics, and the philosophical issues they raise. For this, the only quantifiers needed were ‘all’ and/or ‘some’. Since ‘the’ is convenient, it was useful to define it from the other two, plus identity. Numerical quantifiers [exactly, at least, at most, n Fs], are similarly definable. Being a philosopher-mathematician, Russell had the familiar interest in specifying a minimal conceptual base for doing a maximum amount of intellectual work. Looming above all else was his grand project of reducing mathematics to logic. The denoting phrases relevant to this were few, and it was natural for one with his agenda to define them in terms of a single quantifier ‘∀x’, plus identity.

Doing this guaranteed that no numerical notions were being smuggled in, which is what one wants if one’s aim is to show how we could make sense of all of mathematics on a very slender base of logical primitives. Thus, for someone on Russell’s quest, the following question was not a pressing concern.

Q. Are the propositions yielded by the logicist project merely equivalent to those grasped by ordinary English speakers when they do mathematics, or are the propositions yielded identical with those grasped by ordinary speakers?

Although today we recognize such questions as being important for constructing empirical theories of meaning for natural languages, theories of this sort were not top priorities for Russell or Frege. Despite the fact that their primary logico-mathematical projects laid important parts of the foundation of the scientific study of natural languages, the latter is a recent innovation that didn’t exist then.

Names and Negative Existentials Revisited
As the years went by, Russell’s accelerating restriction of logically proper names to tags for Platonic universals, plus demonstratives referring to one’s own current thoughts and experiences, led him to adopt a view of all ordinary proper names (for people, places, and things) as disguised definite descriptions. On this view, the meaning of an ordinary name, for a speaker s at a time t, is identified with a description s is willing to substitute for it at t. In addition to fitting his doctrines of meaning and acquaintance, this allowed him to adopt a roughly Fregean solution to Frege’s Puzzle. Russell himself recognized this on the next to last page of “On Denoting.”

“The usefulness of identity is explained by the above theory. No one outside a logic-book ever wishes to say “x is x,” and yet assertions of identity are often made in such forms as “Scott was the author of Waverley … The meaning of such propositions cannot be stated without the notion of identity, although they are not simply statements that Scott is identical with another term … The shortest statement of “Scott is the author of Waverley” seems to be : “Scott wrote Waverley; and it is always true of y that if y wrote Waverley, y is identical with Scott”. It is in this way that identity enters into “Scott is the author of Waverley”; and it is owing to such uses that identity is worth affirming.”

Although Russell here uses ‘Scott’ as a proper name to illustrate his point, the logic of his position is that since it may occur with another name in an informative identity statement – one “worth affirming” – it can’t really be a logically proper name. In short ordinary names, which can occur in non-trivial identity sentences, must be disguised descriptions.
So someone who doesn’t know that (11b) and (12b) express truths, simply associates different descriptions with the two names, and so means different things by them, while using the (a) and (b) sentences to express different propositions.

11a. Hesperus is Hesperus.
   b. Hesperus is Phosphorus.

12a. Samuel Clemens is Samuel Clemens.
   b. Samuel Clemens is Mark Twain.

This semantic analysis of ordinary names fit Russell’s solution to the problems posed by negative existentials like (13).

13. Socrates doesn’t exist.

Since (13) is true, he reasoned, ‘Socrates’ doesn’t refer to anything. So, if ‘Socrates’ were a logically proper name, it wouldn’t mean anything. But if it didn’t mean anything, then (13) would be neither true nor meaningful. Since it is both, ‘Socrates’ isn’t a name, but is short for a definite description.

As we have seen, however, examples like these are not so simple. Suppose ‘Socrates’ is short for ‘the teacher of Plato’. Then, (13) means the same as (14a), which Russell would analyze as (14b).

14a. The teacher of Plato doesn’t exist.
   b. ~∃x∀y(y taught Plato ↔ y=x)

What, you may ask, happened to the predicate ‘exist’ in going from (14a) to (14b)? Since the existence claim is already made by the clause expanding the description, there is no need to add ‘& x exists’ to the formula to which the quantifiers are attached. Worse, Russell thought he had discovered that existence and nonexistence claims always involve a quantifier, and that the grammatical predicate ‘exist’ never functions logically as a predicate. In fact, he makes the same three mistakes that Frege seems to have made. The first mistake was holding that existence is the property of propositional functions expressed by the quantifier ‘∃x’. The second was thinking that it is in the nature of that quantifier, and its natural language counterparts, to range only over existing things. The third mistake, was in thinking that ‘exist’ can’t function semantically as predicate true of all and only existing objects.

In fact, we need such a predicate in the logical form (14c), of (14a), which results from our revised analysis of definite descriptions as generalized quantifiers.

14c. ~[the x: x taught Plato] x exists.

This statement is true iff it is not the case that the propositional function p_{Exist} – which assigns to an object o the proposition that o exists – is true of an object that is unique in having taught Plato. More simply, (10c) is true iff it is not the case that an individual who uniquely satisfies ‘x taught Plato’ makes ‘x exists’ true.

Next compare (14) with (15).

15a. The teacher of Plato is dead and so doesn’t exist.
   b. ~[the x: x taught Plato] (x is dead and x exists)
   c. [the x: x taught Plato] (x is dead, and so ~ x exists)

The logical form of (15a) is clearly not (15b), but (15c) – which is true iff an individual who uniquely satisfies ‘x taught Plato’ makes ‘x is dead and so ~ x exists’ true, when assigned as value of ‘x’. For this to be so, the range of the quantifier ‘[the x: x taught Plato]’ – i.e. the range of objects to which the propositional function p_{taught Plato} is applied – must include those who once existed, but no longer do. On this assumption, (15a) and (15c) correctly come out true – since ‘x exists’ is false, and ‘x is dead and so
\[ \sim x \text{ exists} \] is true, when the non-existent Socrates is assigned as value of ‘x’. This is significant because the meanings of variables, relative to assignments, are, in the Russellian scheme, simply the objects assigned to them. But if formulas containing variables can be meaningful and true when assigned referents that no longer exist, then the fact that Socrates no longer exists doesn’t show that ‘Socrates’ doesn’t refer to him. Nor does it show that the referent of ‘Socrates’ isn’t its meaning, or that (13) can’t be both true and meaningful, even if ‘Socrates’ genuinely names Socrates. So, Russell’s historically influential argument to the contrary is inconclusive.

What about true, meaningful negative existentials like (16).

16. Santa Claus doesn’t exist.

The Russellian argument that names of fictional characters like ‘Santa Claus’ must be short for (non-denoting) descriptions is also inconclusive. For if ‘Santa Claus’ were simply short for some such description, then (17a) and (17b) would be obviously and straightforwardly false.

17a. Santa Claus lives at the North Pole and delivers presents on Christmas eve.

b. Santa Claus is a fictional character.

But they aren’t false; both are naturally understood as saying something true. Standardly, someone who uses (17a) does so in order to assert the truth that according to the traditional Christmas story, Santa Claus lives at the North Pole and delivers presents on Christmas eve. By contrast, (17b) is not used to report what is true according to the fiction; rather it is used to state a truth about the fiction, considered as a cultural object. (17b) is true because there really is a Christmas story, the main character of which is named ‘Santa Claus’. Just as books, plays, and stories are real existing things that can be named, so parts of them – acts, scenes, and characters – are such things. ‘Santa Claus’ is the name of one of these things – a character in a story. (17b) is true because the story of which it is a part is a fiction; (17a) is true because it is used to truly report what is true in, or according to, the fiction. In what sense, then, is (16) true? It is true when used to assert that the fictional character Santa Claus does not exist as a real person, with the properties the character is portrayed as having in the story. All of this can be better explained by treating ‘Santa Claus’ as a logically proper name of a real part of an existing story, than by treating it as a disguised description.

This failure of one of the chief negative arguments against treating ordinary proper names as logically proper names is significant. Russell always had a lively sense of the role of names in language as tags which merely label without describing, and he never gave up the category entirely. He did however, end up drastically limiting the scope of naming, being pushed to this position by three main factors – the negative existentials argument, the need for a solution to Frege’s puzzle when ordinary names are involved, and his general internalist epistemology. Neither the argument for negative existentials nor the internalist epistemology is persuasive. Thus the main challenge for neo-Russellians has been to find a non-Fregean solution to Frege’s puzzle that allows us to treat ordinary names as really names. Although this has proven to be very difficult to do, there are now good reasons to think it can be met.