Issues in Estimating New Keynesian Phillips Curves in the Presence of Unknown Structural Change*

Mariano Kulish† and Adrian Pagan‡

November 17, 2012

Contents
1 Introduction 2
2 The Model and Estimators 4
3 A Simple Variant of the Solution Algorithm 5
4 Simulation Experiments 6
  4.1 Parameter Values ........................................... 6
  4.2 The Experiments .............................................. 7
    4.2.1 Experiment 1: No breaks .............................. 7
    4.2.2 Experiment 2: Breaks in Means but not in the Intercept of the Phillips Curve ............................ 8
    4.2.3 Experiment 3 π Breaks, Intercept in Phillips Curve Shifts .............................................. 9
    4.2.4 Experiment 4 x Breaks, Intercept in Phillips Curve Shifts .............................................. 10
    4.2.5 Experiment 5 - Experiment 3 but Agents think Breaks at 60% of Sample .............................. 10
5 Breaks and the Euro Area New Keynesian Phillips Curve 11
6 Conclusion 13
7 References 14
8 Appendix 14

*We thank Sophocles Mavroeidis for comments on an earlier draft.
†School of Economics, University of New South Wales, m.kulish@unsw.edu.au
‡School of Economics, University of Sydney, adrian.pagan@sydney.edu.au
1 Introduction

Many papers which have estimated models with forward looking expectations have reported that the magnitude of the coefficients of the expectations term is very large when compared with the effects coming from past dynamics. This has sometimes been regarded as implausible and led to the feeling that the expectations coefficient is biased upwards. Exactly why that should be the case is less clear. One possibility is that weak instruments can result in an estimator bias in small samples e.g. Mavroeidis (2004), although there is no reason to think it is upwards. Another possibility is specification error in the structural equation containing the expectations. Whilst there is little one can say about this in general, as the nature of the specification error will be crucial, a relatively general argument that has been advanced is that the bias could be due to changes in the means of the variables entering the structural equation. Thus, using Bai-Perron tests to locate breaks in the mean of inflation, Russell et al. (2010) argue that there were 8 breaks in the mean of the U.S. inflation rate over 1960-2007. Taking these to coincide with changes in the intercept of the New Keynesian (NK) Phillips curve they augmented the NK equation with dummy variables that coincided with the breaks. Re-estimation with such dummies then showed that the coefficient of the expectations term was greatly reduced from the situation when breaks were not allowed for. This led them to conclude that (p 1), “Once the shifts in the mean rate of inflation have been accounted for in the estimation of the United States Phillips curves we find that ... there is no significant role for expected inflation in the NK and hybrid models of inflation”.

Castle et al. (2011) provide an explanation of the outcome documented by Russell et al. It revolves around the fact that a standard way of estimating an equation with forward looking expectations is to replace the latter with future observed values, and to then apply an instrumental variables estimator to the resulting equation in observables. When breaks are unaccounted for the future values contain them. Thus Castle et al’s argument is that the explanatory power of the future observables term is due to the breaks and not to forward looking expectations. Technically, one gets an inconsistency in the instrumental variables (IV) estimator of the coefficient of the future expectations term. In some simple experiments they show that this effect can be substantial.

In this paper we investigate the issue of upward bias in the estimated coefficients of the expectations variable based on a model where we can see what causes the breaks and how to control for them. Since many of the applications involve an NK Phillips curve we work with that as the structural equation, embedding it in a simple NK model that also has equations for real marginal cost and an interest rate rule. In each case the agent may know of the breaks but the econometrician doesn’t. We discuss how to solve this model in the presence of breaks, both when agents know exactly when the breaks occur and when they get the time of the break wrong. The method of solution does not depend on the simple model we use for experiments but can be used for any DSGE model. It is set out in detail in Kulish and Pagan (2012).
Because the model is simple we are able to perform an experiment in which there are breaks in the means of the target inflation rate and real marginal cost but these offset one another to produce no breaks in the intercept of the NK Phillips curve. Yet we find a bias in some commonly used estimators. Since the equation is correctly specified, due to the intercept being constant (and in this experiment we assume that agents know exactly the time of the mean shifts), the only reason that this can happen is the use of weak instruments. This leads us to make a distinction between large-sample biases due to specification errors (in the structural equation) and those arising in smaller samples which can come from weak instruments. The latter can sometimes be resolved by using different estimators whereas the former can’t be. We find that breaks in the means of the series can often change the properties of instruments a great deal, and may well be a bigger source of small sample bias than that due to specification error. Moreover we also find that the direction of the specification bias is not predictable. With some estimators and breaks it is the case that the coefficient of the expectations variable is over-estimated, but in others it is under-estimated. This leads one to the conclusion that it is necessary to check for factors such as the presence of weak instruments before deciding that the magnitude of any estimator reflects specification errors coming from breaking means.

The next section sets out our simple model and distinguishes three estimators of the NK Phillips curve. One of these is infeasible but gives a useful benchmark. Section 3 then provides a simplified account of how the NK model is solved in the presence of structural change, while section 4 looks at a range of simulations, beginning with no breaks, moving on to breaks in the reduced form but not the structure, and finishing with breaks in both the structure and reduced form. The estimators introduced in section 2 are examined, and we assess which one performs best in the presence of breaks. In this section we also investigate the robustness of our results to agents not knowing precisely the timing of breaks when they form expectations.

Section 5 looks at the empirical work in Castle et al. (2011) on the NK Phillips curve in the Euro Area. They argue that the structural equation requires the addition of a number of indicator variables which, when added, reduce the estimated expectations coefficient by a large amount. Of course indicators are only very short-lived breaks, whereas the breaks we look at in this paper are rather longer-lived. Nevertheless, even short-lived breaks can cause specification bias, and their presence in the reduced form can lead to weak instruments. We assess whether the smaller coefficient on expectations that Castle et al. (2011) note when dummy variables are present is due to weak instruments or to specification issues. Our finding suggests that it is probably a consequence of weak instruments.
2 The Model and Estimators

Designating $\pi_t$, $x_t$ and $r_t$ as the inflation rate, real marginal cost and the interest rate respectively while $\bar{\pi}_t$, $\bar{x}_t$ and $\bar{r}_t$ are their long-run equilibrium values\(^1\), we will be using the system of equations

\[
\pi_t = \{(1 - s)\bar{\pi}_t - \delta \bar{x}_t\} + \alpha \pi_{t-1} + \gamma \mathbb{E}_t \pi_{t+1} + \delta x_t + \varepsilon_{1t}
\]

\[
x_t = \{(1 - \rho_1)\bar{x}_t - d \times \bar{\pi}_t\} + \rho_1 x_{t-1} + d(r_{t-1} - \pi_{t-1}) + \varepsilon_{2t}
\]

\[
r_t = \{(1 - \lambda_1)\bar{r}_t - \lambda_2 \bar{x}_t - \lambda_3 \bar{\pi}_t\} + \lambda_1 r_{t-1} + \lambda_2 x_t + \lambda_3 \pi_t + \varepsilon_{3t},
\]

where $\bar{r}_t = \bar{\pi}_t + \bar{x}_t$, $\bar{\pi}_t$ is the equilibrium real rate of interest and $s = \alpha + \gamma$. It is necessary to explain how these equations were arrived at. Generally the NK system comes from a set of equations that have been log linearized around steady state values. Thus the NK Phillips curve has the form

\[
\bar{\pi}_t = \alpha \bar{\pi}_{t-1} + \gamma \mathbb{E}_t \bar{\pi}_{t+1} + \delta \bar{x}_t + \varepsilon_{1t},
\]

where $\bar{\pi}_t = \pi_t - \bar{\pi}$ and $\bar{x}_t = x_t - \bar{x}$ are deviations from the steady state values. Now the simple extension of this to a varying equilibria case would seem to be to define $\bar{\pi}_t = \pi_t - \bar{\pi}$ and $\bar{\pi}_{t+1} = \pi_{t+1} - \bar{\pi}_{t+1}$. This is straightforward for $\bar{\pi}_t$ but less so for $\bar{\pi}_{t+1}$. Consider $\mathbb{E}_t (\bar{\pi}_{t+1}) = \mathbb{E}_t (\pi_{t+1} - \bar{\pi}_{t+1})$. At the point where expectations are formed agents think the target inflation rate is $\bar{\pi}_t$ and it is not until $t + 1$ that their beliefs change. So it is correct to use $\mathbb{E}_t (\pi_{t+1} - \bar{\pi}_t)$ rather than $\mathbb{E}_t (\pi_{t+1} - \bar{\pi}_{t+1})$ as the driver of $\bar{\pi}_t$. In a similar vein, what matters to agents when setting $\pi_t$ is how far away the inflation rate at $t - 1$ is from the equilibrium at $t$. Consequently $\pi_{t-1}$ is best defined as $\pi_{t-1} - \bar{\pi}_t$. Using those definitions produces (1). Similar arguments give (2) and (3).

The only two equilibrium values that change in our experiments are for the inflation target ($\bar{\pi}_t$) and real marginal cost ($\bar{x}_t$)\(^2\). It is known that when $\bar{\pi}_t$ and $\bar{x}_t$ vary one might expect the parameters of the NK Phillips curve to be dependent on these quantities, and so changing as well - see Ascari (2004) and Cogley and Sbordone (2008). Although our solution algorithm does allow for the ‘slope’ parameters such as $\alpha$ and $\gamma$ to be indexed by time as well, we want to focus upon how changing equilibrium values affect the estimators of the NK Phillips curve coefficients, as that has been the focus in the literature.

We can think of three possible IV estimators of the Phillips curve. The first, the unrestricted estimator (UE), works with the equation

\[
\pi_t = c_1 + \alpha \pi_{t-1} + \gamma \pi_{t+1} + \delta x_t + \{\gamma (\mathbb{E}_t \pi_{t+1} - \pi_{t+1}) + \varepsilon_{1t}\}.
\]

Variables that are uncorrelated with $\varepsilon_{1t}$ provide instruments for the regressors and these are $\pi_{t-1}, x_t, x_{t-1}$ and $r_{t-1}$. $x_t$ qualifies as an instrument because only lagged values determine it and $\varepsilon_{2t}$ is uncorrelated with $\varepsilon_{1t}$. The appendix shows that $\pi_{t+1}$ depends on $x_t, \pi_{t-1}$ and $r_{t-1}$ but not $x_{t-1}$. Hence $x_{t-1}$ is not

\(^1\)The values to which $\pi_t, x_t$ and $r_t$ would converge in the absence of shocks.

\(^2\)Of course $\bar{\pi}_t$ changes because $\pi_t$ changes.
a relevant instrument for \( \pi_{t+1} \), a fact noted by Pesaran (1987) when \( d \) in (2) is zero. Consequently, \( \pi_{t-1}, x_t \) and \( r_{t-1} \) provide exactly the right number of instruments for the three variables \( \pi_{t-1}, \pi_{t+1} \) and \( x_t \). In much empirical work a broader set of instruments is assumed without specifying a model, but it is useful to have a small model from which to generate the instruments.

A second estimator - the restricted IV estimator (RE) - works with the re-parameterized equation (where \{\} contains the error term).

\[
\pi_t - s \pi_{t-1} = c_1 + \gamma (\pi_{t+1} - \pi_{t-1}) + \delta x_t + \{\gamma (E_t \pi_{t+1} - \pi_{t+1}) + \varepsilon_t\},
\]

where two instruments are needed for \( \pi_{t+1} - \pi_{t-1} \) and \( x_t \). These are found by combining together the three potential ones \( \pi_{t-1}, x_t \) and \( r_{t-1} \).

Finally, there is a third estimator which applies RE assuming that \( s = 1 \). We call this RES. The rationale for RES comes from Gali and Gertler’s (1999) NK model - when the discount rate is .99 the value of \( s \) varies between .99 and .996. This variation comes from changing either the probability of firms resetting prices or the fraction of firms setting optimal prices, over their permissible ranges of zero to one. The RES estimator will therefore be biased in large samples since the error term will be augmented by \((1-s)\pi_{t-1}\), and that will be correlated with the instruments.

3 A Simple Variant of the Solution Algorithm

Kulish and Pagan (2012) set out the algorithm used to compute solutions to the system. It has more general application than the context we are working with here, so we provide a simplified discussion of its workings in order to highlight some significant features of it.

The system we consider has the format

\[
z_t = c_t + A z_{t-1} + B E_t z_{t+1} + \varepsilon_t,
\]

where \( z_t \) is a vector of \( n (=3) \) variables \((\pi_t, x_t, r_t)\), \( c_t \) are the intercepts in the equations, and \( \varepsilon_t \) are i.i.d.\((0, \sigma^2)\) shocks. It is necessary to allow agents to have potentially different beliefs about the timing of any breaks than what the reality is. In order to study the effects of mean breaks we assume that agents always know the parameters \( A \) and \( B \) of the system, and it is only \( c_t \) that their beliefs may be incorrect about. Agents will be taken as believing that the intercepts of the three equations at time \( t \) have the values \( c^a_t \) rather than the true values. Then agents solve the system (4) (with \( c^2_t \) replacing \( c_t \)) to form their expectations \( E_t^a z_{t+1} \), where the "a" indexes the agent’s beliefs. Those expectations will then determine the actual outcomes of the system i.e. the observed variables will be consistent with

\[
z_t = c_t + A z_{t-1} + B E_t^a z_{t+1} + \varepsilon_t.
\]

Of course when \( c^a_t = c_t \), \( E_t z_{t+1} \) and \( E_t^a z_{t+1} \) coincide.

We adopt the Binder and Pesaran (1995) solution method for solving the system. Briefly this involves converting (4) into a purely forward looking form,
solving that, and then recovering the solution to the original system. In the case that agents believe that the economy is described by (4) (with \( c_t = c^a_t \)) the solution method comes down to solving

\[
Z_t = (I - BP)^{-1}c^a_t + S\mathbb{E}_t(Z_{t+1}) + Q\varepsilon_t,
\]

where \( Z_t = z_t - Pz_{t-1}, \) \( S = (I - BP)^{-1}B, Q = (I - BP)^{-1}T \) and \( P \) is chosen such that \((A + BP^2 - P) = 0\). Because shocks \( \varepsilon_t \) are i.i.d. the solution will be

\[
Z_t = \mathbb{E}_t \sum_{j=0}^{\infty} S^j(I - BP)^{-1}c^a_{t+j} + Q\varepsilon_t,
\]

making

\[
z_t = Pz_{t-1} + \sum_{j=0}^{\infty} S^j(I - BP)^{-1}c^a_{t+j} + Q\varepsilon_t.
\]

and so

\[
\mathbb{E}_t^a z_{t+1} = Pz_t + \sum_{j=1}^{\infty} S^j(I - BP)^{-1}c^a_{t+j}.
\]

It is interesting to observe here that, when agents form expectations, they do so by weighting the data \( z_t \) and, as the realized \( z_t \) will incorporate any structural change that has happened, agents’ expectations will partially adapt to the structural change, even when their beliefs are incorrect.

### 4 Simulation Experiments

#### 4.1 Parameter Values

In the experiments we report on later some parameters in the NK model are assumed constant. These are

\[
\begin{align*}
\alpha &= (.7, 3), s = .99, \delta = .1, \rho = .7, \pi = 0, \bar{x} = 0, \\
\bar{\pi} &= .02, d = -.1, \lambda_1 = .7, \lambda_2 = .2, \lambda_3 = .5, \\
\sigma_{\varepsilon_1} &= .001, \sigma_{\varepsilon_2} = .001, \sigma_{\varepsilon_3} = .001
\end{align*}
\]

The structural changes we allow for are in either \( \bar{\pi} \) or \( \bar{x} \). If there are no breaks (experiment 1) these remain zero throughout. In the remaining experiments there is either a break in the inflation target or real marginal cost. When it is the former \( \bar{\pi} \) becomes .02 after forty percent of the sample while, if it is the latter, \( \bar{x} \) becomes -.015 at that point in the sample. Experiment 2 is an exception. Here a break in the inflation target occurs as above, but we allow potential output to move from zero to \( \bar{x}_t = \frac{(1-s)\bar{\pi}_t}{\rho} \). This choice means that the intercept in the NK Phillips curve, \( c_1 \), is zero i.e. it does not show any breaks. Experiments 3 and 4 look at changing either the mean inflation rate or the mean of potential output. Finally, experiment 5 looks at whether our
conclusions depend on agents correctly knowing the timing of any breaks. To assess this we consider the case where agents believe that the means $\bar{x}$, $\bar{\pi}$ remain constant until 60% of the sample is completed, whereas the actual shift is at 40% of the sample. The parameter values above seem fairly standard. $s$ was chosen to be .99 owing to the fact that in NK Phillips curves $s$ must equal or exceed the discount rate. Two values of $\alpha$ are allowed for to reflect either a weak ($\gamma = .99 - \alpha = .29$) or strong ($\gamma = .69$) effect from forward looking expectations.

The six experiments were chosen to elucidate many of the issues mentioned in the introduction to the paper. When examining the results our focus will be on two things. Firstly, even if there is no structural change, we can have a small-sample bias in the estimator of $\gamma$, simply because of weak instruments. Secondly, when the intercept does shift, and no allowance is made for that, a specification error is created, which can cause a large-sample bias. One way to distinguish these two effects is to have breaks in means but none in the NK Phillips curve intercept i.e. they offset as in Experiment 2. Then any bias must be due to weak instruments since the equation is correctly specified.

One way to recognize weak instruments (infeasible in practice) is to compare the mean and median of the estimators, since in many simulations (but not all) weak instruments show up as the IV estimator being non-normally distributed. Another way is to extend the sample and see if the bias goes away. Finally, for the UE (RE), examining the F test for a zero coefficient of $r_{t-1}$ ($r_{t-1}, \pi_{t-1}$) in the regression of $\pi_{t+1}$ ($\pi_{t+1}, \pi_{t-1}$) against $x_t, \pi_{t-1}$ and $r_{t-1}$ will give useful (feasible) information about weak instruments. The popular rule of thumb that $F > 10$ is often used for assessing the quality of the instruments. In interpreting later results it should be noted that structural change not only affects intercepts in equations like the Phillips curve but can also make instruments stronger or weaker due to changes in the reduced form. As mentioned earlier breaks occur at 40% of the sample size and, when agents have the timing wrong, they expect them to have been at 60%. We use percentages, as this enables us to increase the sample size so as to study the “asymptotic” properties, as well as the small sample ones.

4.2 The Experiments

4.2.1 Experiment 1: No breaks

This experiment keeps all parameters constant while $\bar{x}$ and $\bar{\pi}$ are both set to zero. Two sample sizes are used, $T = 100$ and $T = 1000$, and 500 replications are performed to assess estimator bias. The estimators are the unrestricted (UE) and two restricted estimators - one of these uses the correct sum of the forward and backward parameters (RE) and the other just sets it to unity (RES). Table 1 contains the results.

---

3 In fact the bias can exist for very large samples, even though it disappears asymptotically.
A weak instrument bias in the UE shows up strongly here. In the case of weak forward looking expectations ($\gamma = .29$) it is still present when there are 1000 observations. The F tests for instrument quality in the UE case are .72 ($T = 100$) and 10.6 ($T = 1000$). There are two free instruments for the RE and these are much stronger for the variable that needs instrumenting $\pi_{t+1} - \pi_{t-1}$ with the F statistics being 36.4 ($T = 100$) and 457 ($T = 1000$).

It is clear that the performance of the RE deteriorates as $\gamma$ declines. The reason is that inflation becomes more persistent and, since we are effectively looking at the correlation of $\pi_{t+1} - \pi_{t-1}$ with the instrument $\pi_{t-1}$, this declines with $\gamma$. The RES estimator not only has a weak instrument bias but one due to the invalid assumption that $\gamma + \alpha = 1$. For small $\gamma$ this can be sizable, as the $T = 1000$ case shows. If there is actually a weak effect of forward expectations then the RES estimator will suggest that it is stronger than it really is. However, if the expectations effect is strong then the RE and RES biases are quite small. In practice many people have imposed the restriction that $s = 1$ in estimation. It should be noted that the bias could be lower than shown here as we have replaced the true $s = .99$ with an assumed value of $s = 1$, whereas a discount factor of .99 would imply that the true value of $s$ lies much closer to unity than we have assumed in this experiment. Since the bias depends directly on the difference between the assumed and true values it can be much smaller if these are close.

4.2.2 Experiment 2: Breaks in Means but not in the Intercept of the Phillips Curve

Here we allow $\bar{\pi}_t$ to break, going to .02 from zero, but choose $\bar{x}_t$ so that the intercept in the Phillips curve remains constant. All other parameters are as in Experiment 1. As the standard deviation of inflation is .003 this is an enormous break and is chosen to maximize the effects. There is no specification bias in the UE and RE estimators due to the compensating variation in $\bar{x}_t$ and any biases in UE and RE must come from weak instruments. Table 2 shows that the UE estimator behaves in much the same way as it did before, although it may be slightly better. Using the F test the "free" instrument $\tau_{t-1}$ does seem slightly better behaved. This happens due to the break in the inflation process. The RE
performance has weakened because the break in the inflation process has made it much more persistent, and so $\pi_{t-1}$ is a weaker instrument for $\pi_{t+1} - \pi_{t-1}$. This demonstrates an interesting feature: structural breaks may affect the quality of instruments. Indeed they might even become better. It looks as if the RES performs much the same but the bias increases as the sample size rises. When \( T = 5000 \) the mean/median of of $\hat{\gamma}_{RES} = .9$, so that the upward bias in $\hat{\gamma}$ due to assuming $\gamma + \alpha = 1$ seems to be between .15 and .2.

<table>
<thead>
<tr>
<th>Table 2 Estimators of the Phillips Curve, Breaks in Means</th>
<th>but not in the Intercept of the Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T=100 )</td>
<td>True $\gamma = .29$</td>
</tr>
<tr>
<td></td>
<td>UE</td>
</tr>
<tr>
<td>Median</td>
<td>.43</td>
</tr>
<tr>
<td>Mean</td>
<td>.41</td>
</tr>
<tr>
<td>( T=1000 )</td>
<td></td>
</tr>
<tr>
<td>Median</td>
<td>.32</td>
</tr>
<tr>
<td>Mean</td>
<td>.42</td>
</tr>
</tbody>
</table>

4.2.3 Experiment 3 $\pi$ Breaks, Intercept in Phillips Curve Shifts

The experiment now eliminates the offsetting change in $\bar{x}$ of the preceding experiment, resulting in $\bar{x}$ not changing. It should be noted that, even though a large break in $\bar{\pi}$ is allowed ( it becomes .02), this does not lead to a very large break in the Phillips curve intercept, as the latter is $(1-s)\bar{\pi}$. Table 3 gives the results.

| Table 3: Estimators of the Phillips Curve, Break in Inflation Mean | \( T=100 \) | True $\gamma = .29$ | True $\gamma = .69$ |
|-------------------------------------------------------------------|---------------|---------------------|
|                                                                  | UE | RE | RES | UE | RE | RES |
| Median                                                            | .43 | .38 | .46 | .61 | .66 | .67 |
| Mean                                                              | .35 | .37 | .44 | .61 | .69 | .70 |
| \( T=1000 \)                                                      |                                            |                                            |
| Median                                                            | .41 | .24 | .42 | .70 | .69 | .70 |
| Mean                                                              | .39 | .21 | .42 | 1.49 | .69 | .70 |

The instruments for the UE are now better than for the RE ( at \( T = 1000 \) the F test is 9 versus 6.5) when $\gamma = .29$ and, although the instruments are rather weak, the UE results mainly reflect the specification error bias due to the structural change. This is seen by noting that, at \( T = 10000 \), the mean/median of $\hat{\gamma}_{UE} = .4$. UE instruments deteriorate when $\gamma = .69$ and this shows up as slow
convergence to a normal density. At $T = 10000$ the mean/median of $\hat{\gamma}_{UE}$ is .72 (and the mean and median are the same), so the bias due to specification error is actually quite small when there is a strong effect from expectations. The RE (and RES) case is also an amalgam of weak instrument and specification error bias. At $T = 10000$ the mean/median of $\hat{\gamma}_{RE}$ are .2 ($\gamma = .29$) and .39 ($\gamma = .69$), so that the weak instrument bias in smaller samples is apparent. Notice that, in this case, the RE is biassed downwards due to the structural change, and this is also true of RES, showing that breaks in the mean of inflation cannot always be assumed to lead to an upward bias.

4.2.4 Experiment 4 $x$ Breaks, Intercept in Phillips Curve Shifts

The size of the break in $\bar{x}$ is -.015. Again this is some five times the standard deviation and so is very large. Because it is not multiplied by $(1-s)$ in the Phillips curve intercept (as was $\bar{\pi}$) it produces large changes in the latter. Instruments are quite good in this case for UE when $\gamma = .29$, although much better for the RE estimator. When $\gamma = .69$ the UE instrument has an F test of 11.2 when $T = 1000$ while the RE value is 16.2. This shows up in the relative performances. A sample of $T = 1000$ gives estimates that are only a little different from larger samples. Consequently, while there is an upward bias for all estimators (due to structural change) when $\gamma = .29$, there is a downward bias when $\gamma = .69$. Again the bias is relatively small if there are strong expectations effects.

<table>
<thead>
<tr>
<th>Table 4: Estimators of the Phillips Curve, Breaks in Mean of Marginal Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T=100$ True $\gamma = .29$ True $\gamma = .69$</td>
</tr>
<tr>
<td>UE</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Median</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>$T=1000$</td>
</tr>
<tr>
<td>Median</td>
</tr>
<tr>
<td>Mean</td>
</tr>
</tbody>
</table>

4.2.5 Experiment 5 - Experiment 3 but Agents think Breaks at 60% of Sample

By comparing Tables 3 and 5 it is clear that the properties of the estimators are affected by agents mis-timing the structural change but not by a major amount. It seems as if the UE is more affected than RE and RES.
Table 5 Estimators of Phillips Curve, Agents Mis-time Break

<table>
<thead>
<tr>
<th>T=100</th>
<th>True γ = .29</th>
<th>True γ = .69</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UE</td>
<td>RE</td>
</tr>
<tr>
<td>Median</td>
<td>.43</td>
<td>.37</td>
</tr>
<tr>
<td>Mean</td>
<td>-.11</td>
<td>.37</td>
</tr>
<tr>
<td>T=1000</td>
<td>.39</td>
<td>.19</td>
</tr>
<tr>
<td>Mean</td>
<td>.41</td>
<td>.16</td>
</tr>
</tbody>
</table>

All these results seem to suggest that, provided one can find good instruments, the specification bias due to structural change would not be that large when expectations are very important and, even when they aren’t, one wouldn’t find too big a bias. The RES estimator looks attractive if good instruments are available.

5 Breaks and the Euro Area New Keynesian Phillips Curve

Castle et al. (2011) use the Euro Area NK Phillips curve to illustrate the effects of breaks, arguing that the large coefficient on forward expectations is due to them. The analysis of the preceding section has shown that this could be true, but it was also found that biases in the estimators may come from weak instruments. Consequently, we re-do their analysis here, asking whether structural change is likely to be the cause of any large expectations coefficient estimate.

First, we estimate the hybrid NK Phillips curve over the period 1972/2 to 1998/1

\[ \pi_t = c + \alpha \pi_{t-1} + \gamma E_\pi_{t+1} + \delta x_t + \varepsilon_t, \]  

(5)

where \( x_t \) is the real marginal costs (which is labour share, \( s_t \), in Castle et al.’s equations). Their paper states that the instruments used were \( \{ \pi_{t-j} \}^{3}_{j=1}, x_{t-1}, x_{t-2}, gap_{t-1}, gap_{t-2} \), where \( gap_{t-1} \) is the Euro Area output gap. Their estimates of (5) are in (6).

\[ \pi_t = .009 + .28 \pi_{t-1} + .66 E\pi_{t+1} + .01 x_t + \varepsilon_t \]

(6)

(1.8) (2.4) (4.8) (1.8)

They then suggest that some dummy variables be added on to (6) for 1973/1, 1976/2, 1976/3, 1978/4 and 1983/1. Doing that produces

4We are grateful to Ragner Nymoen for providing the data and corresponding with us over its use.
\[ \pi_t = 0.01 + 0.28\pi_{t-1} + 0.65E\pi_{t+1} + 0.01x_t + \text{dummies} + \varepsilon_t, \]  
\text{(7)} \tag{7}

so that the parameter estimates have scarcely changed with the introduction of the dummy variables. How then do they generate a changed coefficient for the forward expectation term? The answer is through an additional regressor - the lagged output gap - whereupon they get \( \hat{\gamma} = 0.18 \). One might argue that the addition of the output gap enables the mark-up to vary with the state of demand, and so the new specification makes sense. But the changed coefficient on expectations comes from such a mis-specification, not from any structural change, if that is what the dummy variables represent. In fact, regressing \( \pi_{t+1} \) on the dummies either by themselves, or in the presence of other regressors such as \( \pi_{t-1} \) and \( \text{gap}_{t-1} \), produces very low F tests for the hypothesis that these have any relationship with \( \pi_{t+1} \). But this is supposed to be the mechanism whereby structural change affects the estimated expectations coefficient. To make this point more forcibly consider experiment 3. Taking T=100 we regress \( \pi_{t+1} \) on a dummy that equals zero for 40% of the sample and unity for the remainder. The F test that the dummy has no impact is 15. One problem is that the dummy variables used in the Euro Area NK Phillips curve seem to be more "blips" than breaks, as they are not sustained, and so have little effect on the expectations coefficient.

We might ask whether imposing the constraint that \( \gamma + \alpha = 1 \) makes a difference? Doing so we find that \( \hat{\gamma}_{RES} = 0.72 \). It should be admitted that the downside for RES here is that neither \( x_t \) nor \( \text{gap}_{t-1} \) are significant, and they have negative estimated coefficients, although if one took a Bayesian perspective and insisted that they be positive, this would not be inconsistent with the data. Nevertheless, it has to be said that the main reason for imposing \( \gamma + \alpha = 1 \) was its potential to improve the quality of instruments a lot, and this was certainly the case in many of the experiments. But, with the Euro data set, instruments are extremely weak for both the UE and RES estimators. It is easy to understand why: inflation is very persistent, with an AR(1) coefficient of 0.98. Consequently, \( \pi_{t-1} \) is by far the biggest contributor to the explanation of \( \pi_{t+1} \). When applying the UE \( \pi_{t-1} \) is effectively not available to instrument \( \pi_{t+1} \) and so the best instrument is excluded. For the RE \( \pi_{t-1} \) is certainly available as an instrument for \( (\pi_{t+1} - \pi_{t-1}) \) but the high persistence means it is a weak instrument. So even though one can say any bias cannot be due to structural change, it may be that there is a high upward bias coming from weak instruments.

At the end of their paper Castle et al. look at the NK Phillips curve estimated over a period that they think has no breaks - 1983/2-1998/1. The idea is to see if one gets a high estimate for the forward coefficient during this era.

---

5 This is sensitive to the instruments used. They decided not to use lagged wage growth as instruments but, if one does, the coefficient becomes 0.27.
They report that $\hat{\gamma}_{\text{UE}}$ is now .082 and so conclude ". confirming that its significance...is as a proxy for unmodeled shifts" i.e. the high value found in the earlier era was due to breaks. It is interesting to note that, if one added in $\Delta w_{t-1}$ and $\Delta w_{t-2}$ (lagged wage inflation) into the instrument set, $\hat{\gamma}_{\text{UE}}$ would be .61. Moreover, $\hat{\gamma}_{\text{RES}} = .70$. So, just as for the "breaks" sample, there is a conflict between the different estimators and we seek to resolve it.

Now for the RES to be trustworthy low persistence in inflation is needed. The post 1983/1 sample satisfies this quite well, as the AR(1) coefficient is .83. We would expect that $\pi_{t-1}$ would be a reasonable instrument for $\pi_{t+1} - \pi_{t-1}$. But other instruments are being used and some of these seem to be ineffective, specifically $\pi_{t-3}, \pi_{t-5}$. These contribute nothing to the explanation of $\pi_{t+1}$ (or even $x_t$). It is not a good policy to utilize weak instruments that are superfluous. So we reduced the instrument set to $\{\pi_{t-j}\}_{j=1}^{5}, x_{t-1}, x_{t-2}, \text{gap}_{t-1}, \text{gap}_{t-2}$.

A second issue is that an instrument for $x_t$ is needed as well as one for $\pi_{t+1}$. Regressing $x_t$ against the instrument set above shows that the dominant explainer (with a t ratio of 5) is $x_{t-1}$. So, if we think of using $x_{t-1}$ as the instrument for $x_t$, this leaves the effective instrument set as $\{\pi_{t-j}\}_{j=1}^{2}, x_{t-1}, x_{t-2}, \text{gap}_{t-1}, \text{gap}_{t-2}$ for RES, and the same for UE, but with $\pi_{t-1}$ deleted. Testing if the RES instruments contribute to $\pi_{t+1} - \pi_t$ we get an F test of 15, so the instruments look satisfactory. In contrast the UE instruments have virtually no association with $\pi_{t+1}$, with the F test being 1.3. Hence the UE will suffer from weak instruments while the RES will be much less affected.

Given this result it is worth getting the RES for the second era using the Castle et al. instrument set, but excluding $\{\pi_{t-j}\}_{j=3}^{5}$. Doing this produces

$$\pi_t - \pi_{t-1} = .006 + .73(\pi_{t+1} - x_{t-1}) + .01x_t - .13\text{gap}_{t-1} + \varepsilon_t, \quad (8)$$

(8) (5.1) (0.8) (0.4)

which is very close to what found with the first era RES. There is serial correlation in the specification however, and that may reflect some mis-specification. Computing Newey-West HAC standard errors one finds that the t ratios are only slightly changed. Thus our assessment would be that the high value for $\hat{\gamma}$ does not come from breaks in the inflation process. It may reflect mis-specification of the curve, although it seems more likely that if there is any "bias" it is coming from weak instruments.

6 Conclusion

Structural change has been conjectured to lead to an upward bias in the estimated forward expectations coefficient in New Keynesian Phillips curves. We have presented a simple New Keynesian model that enables us to assess this proposition. The model enables us to distinguish the effects of specification error caused by structural change from small sample biases that simply arise...
due to weak instruments. Experiments suggest that the latter dominates the former.

Imposing the restriction that the forward and backward coefficients sum to unity seems a useful thing to do, as it generally produces better instruments at the expense of a small specification bias. Interesting findings are that biases are relatively small when the forward coefficient is high and that structural change can actually improve the quality of instruments, so it may actually be beneficial.

We looked at an empirical study of the Euro Area Phillips curve due to Castle et al. (2011) who concluded that the large expectations coefficient was due to structural change. Our analysis suggests that this is not true. It may be that the large coefficient reflects some mis-specification but it is not due to structural change. It seems that the estimators used by Castle et al. were probably subject to weak instrument bias.

7 References


Pesaran, M.H. (1987), The Limits to Rational Expectations, Basil Blackwell

8 Appendix

The system used is (setting means to zero)

\[
\begin{align*}
\pi_t &= \alpha \pi_{t-1} + \gamma E_t \pi_{t+1} + \delta x_t + \varepsilon_{1t} \\
x_t &= \rho_1 x_{t-1} + d(r_{t-1} - \pi_{t-1}) + \varepsilon_{2t} \\
r_t &= \lambda_1 r_{t-1} + \lambda_2 x_t + \lambda_3 \pi_t + \varepsilon_{3t}.
\end{align*}
\]
where the shocks $\varepsilon_{jt}$ are uncorrelated with one another and not autocorrelated. It is known that the solution to this system would have the form (for inflation and potential output)

$$
\pi_t = \phi_1 \pi_{t-1} + \phi_2 x_{t-1} + \phi_3 r_{t-1} + v_{1t}
$$

(12)

$$
x_t = \phi_4 \pi_{t-1} + \phi_5 x_{t-1} + \phi_6 r_{t-1} + v_{2t}.
$$

(13)

Substituting (12) into (9) and taking the expectation we get

$$
\pi_t = \alpha \pi_{t-1} + \gamma (\phi_1 \pi_t + \phi_2 x_t + \phi_3 r_t) + \delta x_t + \varepsilon_{1t}.
$$

(14)

Gathering terms in (14) produces

$$
\pi_t = \psi_1 \pi_{t-1} + \psi_2 x_t + \psi_3 r_t + v_t,
$$

where $\psi_1 = \frac{\alpha}{1 - \gamma \phi}$. Substituting for $r_t$ from (11) we get

$$
\pi_t = \psi_1 \pi_{t-1} + \psi_2 x_t + \psi_3 (\lambda_1 r_{t-1} + \lambda_2 x_t + \lambda_3 \pi_t + \varepsilon_{3t}) + v_t
$$

(15)

$$
= a_1 \pi_{t-1} + a_2 x_t + a_3 r_t + \eta_t,
$$

(16)

where $a_1 = \frac{\psi_1}{\lambda_1 \phi}, a_2 = \frac{\psi_2}{1 - \lambda_2 \phi}, a_3 = \frac{\psi_3 \lambda_3}{1 - \lambda_3 \phi_3}, \eta_t = \frac{\varepsilon_{3t}}{1 - \lambda_3 \phi_3}$. Substituting for $x_{t+1}$ from (13) gives

$$
\pi_{t+1} = a_1 \pi_t + a_2 x_{t+1} + a_3 r_t + \eta_{t+1}.
$$

Substituting from (13) again we get

$$
\pi_{t+1} = a_1 \pi_t + a_2 (\phi_5 x_t + \phi_4 \pi_t + \phi_6 r_t + v_{2t+1}) + a_3 r_t + \eta_{t+1}
$$

$$
= b_1 \pi_t + b_2 x_t + b_3 r_t + \xi_{t+1}
$$

(17)

where $b_1 = a_1 + a_2 \phi_5, b_2 = a_2 \phi_4, b_3 = a_3 + a_2 \phi_6, \xi_{t+1} = \eta_{t+1} + a_2 v_{2t+1}$. Now using (11) for $r_t$ in (17) we get

$$
\pi_{t+1} = b_1 \pi_t + b_2 x_t + b_3 (\lambda_1 r_{t-1} + \lambda_2 x_t + \lambda_3 \pi_t + \varepsilon_{3t}) + \xi_{t+1}
$$

$$
= d_1 \pi_t + d_2 x_t + d_3 r_{t-1} + \zeta_{t+1}
$$

where $d_1 = b_1 + b_3 \lambda_3, d_2 = b_2 + b_3 \lambda_2, d_3 = b_3 \lambda_1, \zeta_{t+1} = \xi_{t+1} + b_3 \varepsilon_{3t}$. Now replace $\pi_t$ by (16)

$$
\pi_{t+1} = d_1 (a_1 \pi_{t-1} + a_2 x_{t-1} + a_3 r_{t-1} + \eta_t) + d_2 x_t + d_3 r_{t-1} + \zeta_{t+1}
$$

$$
= f_1 \pi_{t-1} + f_2 x_t + f_3 r_{t-1} + e_{t+1}
$$

where $f_1 = d_1 a_1, f_2 = d_1 a_2 + d_2, f_3 = d_1 a_3 + d_3$ and $e_{t+1} = d_1 \eta_t + \zeta_{t+1}$. Hence $\pi_{t+1}$ does not depend on $x_{t-1}$ and so the latter is not an instrument.