Problem of the week # 6 (March 7):

We note $X$ the lifetime of a light bulb (time before it breaks), in years. We give the probability density function of the random variable $X$:

$$f(x) = \begin{cases} 
0 & \text{if } x < 0 \\
C e^{-10x} & \text{if } x \geq 0 
\end{cases}$$

where $C$ is a constant to be determined.

(1) Find what should be the constant $C$ so that $f(x)$ is a probability density function.

(2) Compute the cumulative distribution function of $X$. What is $P(X > 10)$?

(3) Compute the expectation of $X$.

(4) Compute the conditional probability $P(X > 20 \mid X > 10)$. 