Please read all of the following rules carefully before proceeding.

- Check that this Exam contains 11 pages.

- Unless otherwise instructed, please clearly indicate all work involved in the solution of each problem. You will receive partial credit for partial progress toward a solution.

- You may use one 8 x 11 in. sheet of paper with notes (both sides); you may not refer to any other books or notes during the course of the exam.

- You may not use a calculator on the exam.

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Total 100

Please encircle the name of your instructor:

A. Asok    N. Emerson    L. Goldstein    W. Hu    Y. Lin
Problem 1 (10 pts.)

Evaluate the integral

\[ \int x^2 \sin^{-1} x \, dx. \]
Problem 2 (10 pts.)

Evaluate the following integral:

\[ \int \frac{3x^2 + x + 1}{x(x^2 + 1)} \, dx. \]
Problem 3 (10 pts.)

Evaluate the following integrals.

a) \( \int_0^{\infty} x e^{-2x} \, dx \).

b) \( \int \frac{dx}{(3 - x^2)^{3/2}} \).
Problem 4 (10 pts.)

Let $\mathcal{R}$ be the bounded region between the graphs of $x = -y^2 + 1$, $x = 0$ and $y = 0$. Set up, but DO NOT EVALUATE, integrals representing the volumes for the solids obtained by rotating $\mathcal{R}$ about:

a) the line $x = 0$;

b) the line $y = -1$. 
Problem 5 (10 pts.)

Consider the function

\[ f(x) = \int_0^x \sqrt{e^t - 1} \, dt. \]

a) Find the length of the curve \( y = f(x) \) for \( 1 \leq x \leq 2 \).

b) Set up the integral to find the area of the surface obtained by rotating the curve \( y = f(x), 1 \leq x \leq 2 \) about the \( x \)-axis. DO NOT EVALUATE THE INTEGRAL.
Problem 6 (10 pts.)

(Differential Equations) Initially, a tank contains 100 gallons of pure water. Starting at time $t = 0$, a brine solution of concentration 0.2 lbs salt/gallon is added to the tank at a rate of 3 gallons/minute. As the brine solution is added, the tank is mixed and drained at a rate of 3 gallons/minute.

a) Write a differential equation for the amount of salt the tank contains at time $t$.

b) Solve the differential equation in Part (a).

c) How long does it take for the concentration of salt in the tank to reach 0.1 lbs/gal?
Problem 7 (10 pts.)

Determine whether each series is absolutely convergent, conditionally convergent or divergent; state precisely which convergence test(s) you use to determine your answer.

a) \[ \sum_{n=0}^{\infty} \frac{2^{3n} - 4}{11^n}. \]

b) \[ \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n} + 2}. \]

c) \[ \sum_{n=0}^{\infty} \left( \frac{n^3 + n^2 + 2}{3n^3 - n^2 + 1} \right)^{2n}. \]
Consider the function \( f(x) = \sinh^{-1} x \); recall that \( f'(x) = \frac{1}{\sqrt{1+x^2}} \).

Below, when you are asked to write down a series, please give explicitly the first 3 non-zero terms as well as the general term.

a) Write down a Taylor series expansion for \( f(x) \) about the point \( x = 0 \).

b) Determine the radius of convergence of this Taylor series.

c) Determine the value of \( f^{(31)}(0) \).
Problem 9 (10 pts.)

Throughout this problem, set \( f(x) = e^{-x^2} \).

Below, when you are asked to write down a series, please give explicitly the first 3 non-zero terms as well as the general term.

a) Write down the Maclaurin series for \( f(x) \).

b) Use the above series to find a series representation of \( \int_0^1 e^{-x^2} \, dx \).

c) Determine how many terms from the series in Part (b) are necessary if we want to compute the integral with an error of at most 1/1000.
Problem 10 (10 pts.)

Consider one leaf of the four leaf rose, \( r = \cos(2\theta), \ -\pi/4 \leq \theta \leq \pi/4. \)

a) Determine the values of \( \theta \) for which this curve has a horizontal tangent (Hint: first write down an equation whose solutions govern points with horizontal tangents and then use double angle and Pythagorean identities to solve this equation).

b) Determine the values of \( \theta \) for which this curve has a vertical tangent (Refer to the hint given in Part (a)).

c) Find the area of this (one) leaf.