Math 118 Final Exam  
May 5, 2011

Directions. Fill out your name, signature and student ID number on the lines below right now, before starting the exam! Also, check the box next to the class for which you are registered.

You must show all your work and justify your methods to obtain full credit. Write your final answers in the boxes provided. Simplify your answers. Any fraction should be written in lowest terms. You need not evaluate expressions such as ln 5, $e^{0.7}$, and $\sqrt{3}$. Do not use scratch paper; use the back of the previous page if additional room is needed. No calculators are allowed, but you may use the sheet of notes that you brought with you. This may be no more than one sheet of $8\frac{1}{2} \times 11$ paper. You may have anything written on it (on both sides), but it must be written in your own handwriting. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes “straying eyes” and failing to stop writing when told to do so at the end of the exam.

Name (please print):

Signature:

Student ID:

☐ 10–10:50 MWF (Daoust-Ritz)  ☐ 11–11:50 MWF (Jedwab)
☐ 12–12:50 MWF (Jedwab)  ☐ 2–3:20 MW (Daoust-Ritz)

Do not write on this page below this line!

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100 points total
Problem 1. Calculate the following limits. If the limit is infinite, indicate whether it is $+\infty$ or $-\infty$.

a) \( \lim_{x \to 3^+} \frac{x^2 + x - 2}{x^2 - 4x + 3} \)

b) \( \lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 4x + 3} \)

c) \( \lim_{x \to 16} \frac{x - 16}{\sqrt{x - 4}} \)
Problem 2. Find an equation of the tangent line to the curve \( y^2 + x^2 = 5 \) at the point where \( x = 2 \) and \( y \) is negative.
Problem 3. Find the area between the curves $y = x^3$ and $y = 4x$. 
Problem 4. Evaluate the following integrals:

a) \[ \int_1^e x \ln x \, dx \]

b) \[ \int_1^e \frac{1}{x \ln x} \, dx \]

c) \[ \int \frac{x^3}{\sqrt{x^2 + 1}} \, dx \]
Problem 5. Tommy is going to build a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will maximize the total area of the pen? Make sure you justify why your answer is a maximum.
Problem 6. Consider the function \( f(x) = \frac{e^x}{x} \) whose first and second derivatives are \( f'(x) = \frac{e^x(x - 1)}{x^2} \) and \( f''(x) = \frac{e^x(x^2 - 2x + 2)}{x^3} \). Fill in all of the information about the graph of \( f \) below. Do not leave anything blank. If there are none, write ‘none.’

a) Critical numbers of \( f \)

b) Intervals where \( f \) is increasing

c) \( x \)-coordinates of relative maxima, if any

d) Intervals where \( f \) is concave up

e) \( x \)-coordinates of points of inflection

f) Vertical asymptotes
Problem 7. The daily output at a certain factory is $Q(L) = 300L^{2/3}$ units, where $L$ denotes the size of the labor force measured in worker-hours. Currently, 512 worker-hours of labor are used each day. Estimate the number of additional worker-hours of labor that will be needed to increase daily output by 12.5 units. (Note: $2^9 = 512$)
Problem 8. Let \( f(x, y) = e^{x^2 y + 3y} \).

a) Find \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial y} \).

b) If \( x = \frac{1}{t} \) and \( y = t \), use the Chain Rule to find \( \frac{dz}{dt} \) at \( t = 1 \).
Problem 9. Let $f(x, y) = (x^2 + y^2)e^{-x}$.

a) Find all the critical points of $f(x, y)$.

b) Classify each critical point found in part (a) as a relative maximum, relative minimum, or a saddle point.

Relative min at: ____________

Relative max at: ____________

Saddle point at: ____________
Problem 10. Let $f(x, y) = kx^2y$, where $k$ is a constant. Find a value of $k$ for which $\iint_{R} f(x, y) dA = 1$, where $R$ is the rectangular region $0 \leq x \leq 2, 0 \leq y \leq 3$. 