Math 118 Final Exam
May 8, 2008

Directions. Fill out your name, signature and student ID number on the lines below right now, before starting the exam! Also, check the box next to the class for which you are registered.

You must show all your work and justify your methods to obtain full credit. Write your final answers in the boxes provided. Simplify your answers. Any fraction should be written in lowest terms. You need not evaluate expressions such as \( \ln 5 \), \( e^{0.7} \), and \( \sqrt{3} \). Do not use scratch paper; use the back of the previous page if additional room is needed. No calculators are allowed, but you may use the sheet of notes that you brought with you. This may be no more than one sheet of \( 8 \frac{1}{2} \times 11 \) paper. You may have anything written on it (on both sides), but it must be written in your own handwriting. Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes “straying eyes” and failing to stop writing when told to do so at the end of the exam.

Name (please print):

Signature:

Student ID:

- [ ] 10–11 MWF (Voineagu)
- [ ] 10–12 MW (Haskell)
- [ ] 11–12 MWF (Kim)
- [ ] 10–12 TTh (Haskell)
- [ ] 1–2 MWF (Vorel)

Do not write on this page below this line!

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200 points total
Problem 1.

a) Calculate the following limits. If the limit is infinite, indicate whether it is $+\infty$ or $-\infty$.

i) \[ \lim_{x \to -\infty} \frac{x^2 + 2x + 3}{2x^2 + x + 1} \]

ii) \[ \lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{x^2 + x} \right) \]

iii) \[ \lim_{x \to e^+} \frac{9x}{1 - \ln x} \]

iv) \[ \lim_{t \to 5} \frac{\sqrt{2t - 1} - 3}{t - 5} \]

b) For what value(s) of $c$ is the function $f$, given below, continuous at $x = 1$? Explain.

\[ f(x) = \begin{cases} 
  x + c & x < 1 \\
  c^2x + 1 & x \geq 1 
\end{cases} \]
Problem 2. Consider the curve given by the equation

\[ 4e^{2x+y} = x^2y + y. \]

Find the equation of the tangent line to the curve at the point \((-1, 2)\).

Eqn of tangent line:
Problem 3. Consider the function $f$ given by

$$f(x) = \frac{x}{x^2 + 4}.$$  

Its first and second derivatives are shown below (you do not need to derive these yourself).

$$f'(x) = \frac{4 - x^2}{(x^2 + 4)^2} \quad f''(x) = \frac{2x(x^2 - 12)}{(x^2 + 4)^3}$$

Fill in all of the information about the graph of $f$ in the table below. Don’t leave any entries blank; instead, write ‘none’ if appropriate. Then, sketch the graph of $f$. All of the information in the table should be clearly visible on your graph. Remember to show your work.

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<thead>
<tr>
<th>Intervals where $f$ is increasing</th>
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<tr>
<td>Critical numbers of $f$</td>
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<td>Intervals where $f$ is concave up</td>
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<td>$x$-coordinates of points of inflection</td>
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<td>$x$-intercepts</td>
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<td>$y$-intercepts</td>
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<td>Vertical asymptotes</td>
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<td>Horizontal asymptotes</td>
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Problem 4. Consider the function $f$ given by $f(x) = \frac{x + 2 \ln x}{x}$.

a) Find the equation of the tangent line to $f$ at $x = 1$.

b) Using approximation by increments, estimate the value of $f(1.3)$.

c) Find the absolute maximum and minimum of $f$ on the interval $[1, e^2]$ and state the corresponding value of $x$ where each is attained.
Problem 5. A closed cylindrical can with volume $2\pi$ is to be constructed in such a way that its surface area is minimal. Find its dimensions. Remember to show how you know that the surface area is a minimum.
Problem 6. Evaluate the definite and indefinite integrals.

a) \[ \int_1^2 \frac{6x^2 + x - 4}{x} \, dx. \]

b) \[ \int_0^{\ln 2} \frac{e^{2t}}{e^t + 1} \, dt. \]

c) \[ \int x(\ln x)^2 \, dx. \]
Problem 7. In the year 2000, the average American’s salary was $38,100 and it was estimated that this would increase at the rate of

\[ r(t) = 40e^{0.05t} \]
dollars per year, where \( t \) denotes the number of years since 2000. If this is correct, what will the average American’s salary be in the year 2010?

Salary in 2010:
Problem 8. Alex and Betty both go for a one-hour run. The graphs below show their speeds in miles per hour $t$ hours after setting out ($0 \leq t \leq 1$).

a) Who ran the furthest? Circle your answer and explain.

Alex  Betty

b) How far did Alex run?

Distance in miles:
Problem 9. Using $x$ skilled workers and $y$ unskilled workers, a company can produce

$$Q(x, y) = 60x^{1/3}y^{2/3}$$

units per day. Currently the company employs 10 skilled workers and 80 unskilled workers, but it is planning to increase the level of skilled labor by one worker and to decrease the level of unskilled labor by two workers. Use calculus to estimate the effect of these changes on production; will the production level increase or decrease and by approximately how much?

1. (Circle one) Production level will:  
   increase  decrease

2. (Fill in the blank) Production will change by approximately __________ units.
Problem 10. A company produces $x$ units of product A and $y$ units of product B. The selling price of each product depends on the amount produced. More specifically, product A can be sold for $p = 20 - 5x$ dollars per unit and product B can be sold for $q = 4 - 2y$ dollars per unit. The cost $C$ (in dollars) of producing A and B also depends on the amount produced and is given by $C(x, y) = 2xy + 4$. How much should the company produce of A and B in order to maximize their profit? Be sure to justify that your answer is at least a local maximum.

Hint: Profit is equal to total income from sales of A and B minus cost.
Problem 11. Consider the function \( f \) given by

\[ f(x, y) = \frac{\ln(\sqrt{x})}{xy}. \]

Find the volume of the solid that lies under the graph of this function and over the rectangular region \( R \) that is given by

\[ 1 \leq x \leq e, \quad 1 \leq y \leq e^2. \]