Name (please print): ________________________________________________________________

Signature: _______________________________________________________________________

Student ID: _____________________________________________________________________

Directions. Fill out your name, signature and student ID number on the lines above right now before starting the exam! Also, check the box next to the class for which you are registered.

• You must show all your work and justify your methods to obtain full credit. Write your final answers in the boxes provided.

• Simplify your answers. Any fraction should be written in lowest terms. You need not evaluate expressions such as \( \ln 5, e^{i\pi} \) or \( \sqrt{118} \).

• Do not use scratch paper; use the back of the previous page if additional room is needed.

• No calculators are allowed. Turn off your cell phone.

• You may use the sheet of notes that you brought with you, this may be no more than one sheet of \( 8\frac{1}{2} \times 11 \) paper. You may have anything written on it (on both sides), but it must be written in your own handwriting.

• Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes “straying eyes” and failing to stop writing when told to do so at the end of the exam.

Lin (9–10 MWF)  Emerson (11–12 MWF)  Lytvak (12–1 MWF)
Lin (10–11 MWF)  Emerson (12–1 MW)  Zuev (1–2 MWF)
Lin (11–12 MWF)  Schumitzky (2–3:15 MW)

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200 points total
Problem 1 (15 points). Calculate each of the following limits or show that it does not exist. If the limit is infinite, indicated whether it is $+\infty$ or $-\infty$.

(a) $\lim_{x \to -\infty} \frac{1 + 2x + 3x^3}{4 - 5x^3}$.

(b) $\lim_{x \to 0^+} \frac{\sqrt{x + 25} - 5}{x}$.

(c) $\lim_{x \to 0^+} e^{-1/x}$.
Problem 2 (20 points).

(a) Let $x$ and $y$ satisfy the equation: $14 = x^3 + 2xy^2 + 2y^3$. Find $\frac{dy}{dx}$.

\[
\frac{dy}{dx} =
\]

(b) The output $Q$ at a certain factory is related to the inputs $x$ and $y$ by the equation

\[ Q = x^3 + 2xy^2 + 2y^3. \]

If the current levels of input are $x = 2$ and $y = 1$, use calculus and approximation by increments to estimate the change in $y$ required to offset an increase of $x$ by 0.1, so that output $Q$ will remain constant.

$\Delta y =$
Problem 3 (30 points). Consider the function $f(x)$ below. Its first and second derivatives are also given. Fill in all of the information about the graph of $f(x)$ below. Do not leave anything blank. If there are none, write “none.”

$$f(x) = 8\frac{1-x}{(x+1)^2}, \quad f'(x) = 8\frac{x-3}{(x+1)^3}, \quad f''(x) = 16\frac{5-x}{(x+1)^4}, \quad \text{for } x \neq -1.$$

(a) Vertical asymptotes.

(b) Horizontal asymptotes.

(c) Critical numbers of $f(x)$.

(d) Intervals where $f(x)$ is increasing.

(e) The $x$-coordinates of relative minima, if any.
Problem 3 (continued).

\[ f(x) = 8 \frac{1-x}{(x+1)^2}, \quad f'(x) = 8 \frac{x-3}{(x+1)^3}, \quad f''(x) = 16 \frac{5-x}{(x+1)^4}, \quad \text{for } x \neq -1. \]

(f) Intervals where \( f(x) \) is concave up.

(g) The \( x \)-coordinates of points of inflection, if any.

(h) Sketch the graph of \( f(x) \). Your graph should clearly show all the information you found in parts (a)–(g).
Problem 4 (20 points). Find the absolute maximum and the absolute minimum values (if any) of

\[ f(x) = -2x^3 + 3x^2 + 12x - 5 \]

on the interval \(0 \leq x \leq 3\).
Problem 5 (20 points). An LA Galaxy team store can obtain soccer balls with David Beckham’s autograph. The demand for the autographed balls is given by \( q = 100e^{-p/50} \) where \( p \) is the price of a ball in dollars.

(a) Find the price for the autographed balls that will maximize revenue. Recall revenue is price times demand.

(b) Briefly justify why the price you found in fact gives the maximum revenue.
Problem 6 (20 points). A bank offers a choice of two types of savings accounts: a traditional savings account or a Bonus Bucks savings account. Suppose that a customer has $100 to invest.

(a) The traditional savings account pays an annual interest rate of 3% compounded continuously. Find a function for the amount in the traditional account $t$ years after it is opened.

(b) The Bonus Bucks savings account gives a bonus of $10 that is immediately deposited into the account, and then pays an annual interest rate of 1% compounded continuously. Find a function for the amount in the Bonus Bucks account $t$ years after it is opened.

(c) Assuming no other deposits or withdrawals are made, how long does it take for the amount in the traditional account to exceed the amount in the Bonus Bucks account?
Problem 7 (20 points). Consider the region $R$ bounded by the curves $y = e^x$, $y = e^{-x}$, and the line $x = \ln 3$.

(a) Find the $x$-coordinate of the intersection of the curves $y = e^x$ and $y = e^{-x}$.

(b) Sketch the region $R$. Include sufficient detail to set up the definite integral to determine the area of $R$.

(c) Find the area of $R$. 


Problem 8 (15 points). Evaluate the following integrals:

(a) \( \int \frac{\sqrt{x} + xe^{2x} + 5}{x} \, dx \).

(b) \( \int_{1}^{3} \frac{x}{\sqrt{4x - 3}} \, dx \).

(c) \( \int (t - 1)e^{1-t} \, dt \).
Problem 9 (20 points). Let \( f(x, y) = (x - y^2) \ln x \) for \( x > 0 \).

(a) Find all the critical points of \( f(x, y) \).

(b) Classify each critical point you found in part (a) as a relative minimum, a relative maximum, or a saddle point.
Problem 10 (20 points). Evaluate the double integral
\[ \iint_R \frac{y^{1/2} + y^2 e^{xy}}{y} \, dA \]
where \( R \) is the rectangle given by \( 0 \leq x \leq 1 \) and \( 1 \leq y \leq 4 \).