Directions. Fill out your name, signature and student ID number on the lines below right now, before starting the exam! Also, check the box next to the class for which you are registered.

You must show all your work and justify your methods to obtain full credit. Simplify your answers. Any fraction should be written in lowest terms. You need not evaluate expressions such as $\ln 5$, $e^{0.7}$, and $\sqrt{3}$. Do not use scratch paper; use the back of the previous page if additional room is needed. No calculators are allowed. You may use one 8.5" × 11" sheet of handwritten notes (written on both sides). Remember, USC considers cheating to be a serious offense; the minimum penalty is failure for the course. Cheating includes “straying eyes” and failing to stop writing when told to do so at the end of the exam.

Name (please print):

Signature:

Student ID:

☐ 10–12 MW (Haskell) ☐ 12–2 MW (Schumitzky) ☐ 12–2 TTh (Cetin)
☐ 10–12 MW (Zygouras) ☐ 2–4 MW (Blazek) ☐ 2–4 TTh (Macias)
☐ 12–2 MW (Haskell) ☐ 10–12 TTh (Cetin)

Do not write on this page below this line!

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200 points total
Problem 1. Calculate the following limits. If the limit is infinite, indicate whether it is $+\infty$ or $-\infty$.

a) $\lim_{x \to 2^-} \frac{x}{x - 2}$

b) $\lim_{x \to 2^-} \frac{x^4 - 4x^2}{\sqrt{x + 2} - 2}$

c) $\lim_{x \to +\infty} \ln \left( \frac{1}{1 + x} \right)$

d) $\lim_{t \to +\infty} \frac{5}{4 + 2e^{-3t}}$
Problem 2. Consider the function

\[ f(x) = \begin{cases} 
4x + 1 & x < 2 \\
\frac{x^3}{3} & x \geq 2
\end{cases} \]

a) Find \( f(3) \).

b) Find \( \lim_{{x \to 2^-}} f(x) \).

c) Is \( f \) continuous at \( x = 2 \)? Circle yes or no and explain.

YES NO

d) Is \( f \) differentiable at \( x = 2 \)? Circle yes or no and explain.

YES NO
Problem 3. Lake Elysia is situated in the center of the city Rhodendra. The level of copper in the lake $C$ (measured in milligrams per liter) depends on the population of the city $p$ (measured in thousands of people). In other words $C = f(p)$. At present there are 200 thousand people living in the city and the level of copper in the lake is 0.95 milligrams per liter. City officials estimate that $f'(200) = 0.002$. Moreover, they suspect that the graph of $f$ is increasing and concave down as shown in the sketch below.

![Graph of $C = f(p)$](image)

a) Using approximation by increments, estimate what the level of copper in the lake will be when the population of the city reaches 210 thousand people. Remember to include units in your answer.

b) If the population is increasing at the rate of 5 thousand people per year, how fast is the level of copper in the lake increasing? Remember to include units in your answer.
Problem 4. Consider the curve defined by the equation $e^{xy+1} = x$.

a) Use implicit differentiation to find the slope of the tangent line at the point $(1, -1)$.

b) What is the equation of the tangent line at the point $(1, -1)$?
Problem 5. The first derivative $f'$ of a function $f$ is given by

$$f'(x) = 2(x - 1)(x - 2).$$

a) Find all the intervals on which $f$ is increasing. Explain.

b) Find all the critical numbers of $f$ and classify each one as a relative maximum, a relative minimum, or neither.

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You may not need all of the lines.

c) Find all the intervals on which $f$ is concave down. Explain.

d) Find the $x$-coordinates of all points of inflection of $f$. 
Problem 6. A closed box with a square base is to have a volume of 250m$^3$. The material for the top and bottom of the box costs $6 per square meter and the material for the sides costs $3 per square meter.

a) Find the dimensions of the box that costs the least to build. Explain how you know that it costs less than any other box.

b) Can the box be built for less than $1000? Circle yes or no and explain.

YES  NO
Problem 7.

a) Evaluate the integral $\int_{0}^{2} 4xe^{x^2} \, dx$.

b) Evaluate the integral $\int 4x \ln \sqrt{x} \, dx$. 
Problem 8. Let \( f(x) = 2x + 1 \) and \( g(x) = x^2 + x - 1 \).

a) Find the \( x \)-coordinates of all points of intersection of the graphs of \( f \) and \( g \).

b) Find the area that is bounded between the curves \( y = f(x) \) and \( y = g(x) \).
Problem 9. Suppose the profit $P$ of a company selling $x$ units of one product and $y$ units of another product is given by:

$$P = 1000 + 1200x + 900y - (4x^2 + xy + y^2).$$

Find the values of $x$ and $y$ that maximise the profit. Justify that your answer is at least a relative maximum.
Problem 10. Evaluate the double integral $\int \int_R xe^{x+y} \, dA$ where $R$ is the region given by $0 \leq x \leq 1$ and $\ln 2 \leq y \leq \ln 3$. 

$\int \int_R xe^{x+y} \, dA =$