Abstract: In markets with transaction costs, when capital requirements can be made in a basket of currencies or assets, risk measures are naturally set-valued functions. In the dynamic and multivariate setting, the appropriate time consistency property appears to be multi-portfolio time consistency. This is equivalent to the recursive formulation, an additive property for the acceptance sets, the co-cycle condition for penalty functions (for convex risk measures), a version of m-stability (for coherent risk measures), and a super-martingale property (for convex and coherent risk measures). We can additionally use these properties to define multi-portfolio time consistent version of well-known risk measures. By utilizing this time consistency property, we can compute the set of capital requirements at each time and state recursively backwards in time along the event tree. We give conditions under which the backwards calculation of the sets reduces to solving a sequence of linear and convex vector optimization problems. In doing so we motivate why the proposed
procedure can be seen as a set-valued Bellman's principle.