Abstract:

In many incomplete markets, the super replication price of a given (eventually non Markovian) claim rewrites in terms of the minimal super-solution of a well-chosen Backward stochastic differential equation (BSDE). The price obtained is often numerically very high and hereby useless in practice. In order to lower the price, one must accept to take some risk and this can be formalized via the use of quantile hedging type objectives, where the agent only wishes to upper hedge the claim of interest with a given a priori probability of success \( p \). Adding the 'probability of success' process as a new forward process, we are able to revisit quantile hedging type problems in a dynamically consistent manner. This leads to the introduction of so called "BSDEs with weak terminal condition", where the terminal condition of the BSDE is not known explicitly but must only satisfy a given constraint written in terms of expectation. We also introduce extensions of such BSDEs by considering BSDEs with weak or mean reflection, where such type of constraint is imposed on any date \( t \) smaller than \( T \). Connections with 2nd order BSDES will be highlighted. This presentation is based on joint works with Bruno Bouchard, Philippe Briand and Anthony Reveillac.