1. The lifespan of a queen ant follows the following distribution:

\[
\begin{align*}
f(x) &= \begin{cases} 
-\frac{4}{5}x + 1 & 0 \leq x < 1 \\
 0 & 1 \leq x \leq 3 \\
 0 & x < 0, x > 3
\end{cases}
\end{align*}
\]

(a) Find the function \( f(x) = c \) that is distributed \( 1 \leq x \leq 3 \) (note: \( c \) is a constant).

(b) What is the average lifespan of a queen ant?

(c) If after one year, the queen ant is still alive, what is the probability it will still be alive for at least another five months?

(d) When one queen ant dies, another queen ant immediately takes its place, as any given ant hill will always be ruled by a queen ant. If an ant hill goes through 30 different queen ants, what is the probability that this ant hill is at least 28 years and four months old?
2. Slick Motor Tire Inc. uses a tire mold which occasionally, and randomly, produces out-of-round tires. In fact 20% of the tires which it produces are out-of-round. Suppose Slick takes a random sample of 10 tires produced by this tire mold.

   (a) Find the expected number of out-of-round tires among these 10.

   (b) Find the probability that two or more of these 10 tires are out-of-round.

   (c) Suppose now that the Fraud Motor Company buys 120 of the tires produced by this tire mold. Find the approximate probability that between 20 and 30 (inclusive) of these tires are out-of-round.

3. John regularly plays a target shooting competition in his yard. He is not very good at the game and his score has the following probability distribution

<table>
<thead>
<tr>
<th>Score</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob.</td>
<td>0.1</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

   (a) Find the expected value and standard deviation of John’s score.

   (b) Suppose he plays the game twice. Find the probability that his total is at least 5.

   (c) Suppose he plays the game 50 times. Find the probability that his total is at least 75.
4. Happy Hungry Hamburger restaurant cooks an average of 1800 hamburgers patties a week, with a standard deviation of 200 hamburger patties. Assume that the number of hamburger patties grilled follows a normal distribution. A local report states that next week’s demand for hamburger patties will be 2200.

(a) What is the probability that Happy Hungry Hamburger will not be able to meet demand?

(b) This week, Happy Hungry Hamburger grilled an amount of hamburgers that ranks in the top 5% of weeks of amount of hamburgers grilled. How many patties did Happy Hungry Hamburgers grill, at least?

(c) What is the probability that the average number of hamburger patties grilled each week during a 15 week period is at least 1830?

(d) A statistics professor made the following statement on the MATH 218 website: “The probability that Happy Hungry Hamburgers will cook at least 39,000 patties in the next $\$#@!$ weeks will be a small chance of 9.51%” Unfortunately there was a typo in the sentence. How many weeks should that statement have read?

5. Let $X$ be the traveling time from home to work for a randomly chosen office worker in Los Angeles. Assume that $X$ has a normal distribution. A sample of 6 observations from this population gave the following times (in minutes):

$$35 \ 20 \ 25 \ 15 \ 22 \ 21$$

(a) Find point estimates for the population mean $\mu$ and population variance $\sigma^2$. 
(b) Find a point estimate for the standard deviation of the sample mean $X$.

(c) Find a 95% confidence interval for $\mu$.

(d) Suppose that you are told that $\sigma = 8$. Using this additional information find a 95% confidence interval for $\mu$.

(e) Related to part d), how large should the sample be in order to estimate $\mu$ to within 4 units with 95% confidence?

6. A young baseball fan notices that professional baseball players are either right-handed or left-handed.

(a) This young baseball fan would like to determine a 90% confidence interval for the true proportion of baseball players who are left-handed using a sample of 75 players. What would this young baseball fan estimate?

(b) Suppose that in a sample of 75 professional baseball players, 24 are left-handed. Determine the new 90% confidence interval for $p$.

(c) Based on the preliminary sample in part b), determine how many more professional baseball players are needed to estimate $p$ to within +/-0.05?
7. The American Cheese Company sells packages of cheese. These packages are stamped with an expiration date; those packages not sold by the expiration date are discarded. The company claims that on average, the cheese can be kept for at least 14 days past the expiration date before going bad. The government believes that cheese cannot be kept 14 days past its expiration. To test this claim, government inspectors decided to keep 30 packages and record how long past its expiration date it takes each package to go bad. Government inspectors found that the average time it took the 30 packages to go bad was 13 days. The standard deviation $\sigma$ is known to be 2.1 days.

(a) Formulate the null and alternative hypotheses for this test.

(b) Which test statistic should be used to test these hypotheses? Evaluate its numerical value.

(c) Find the P-value.

(d) Based on the P-value, at which significance level(s) should the inspectors reject the null hypothesis? Circle all that apply.

<table>
<thead>
<tr>
<th>Significance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1%</td>
</tr>
<tr>
<td>0.5%</td>
</tr>
<tr>
<td>1%</td>
</tr>
<tr>
<td>5%</td>
</tr>
<tr>
<td>10%</td>
</tr>
</tbody>
</table>

(e) What is the probability that the inspectors wrongly concluded that the cheese cannot be kept for at least 14 days past its expiration if the level of significance is set at 5%?

8. A financial analyst with a major brokerage house specializes in a group of technology stocks. The average price-to-earnings (P/E) ratio of such stocks has been 35. We will assume that the distribution of P/E ratios is approximately normal. The analyst is interested to see if the P/E ratio has changed after the recent activity in the stock market. The population standard deviation is unknown.

(a) State the appropriate null and alternative hypotheses.

(b) Choose a test statistic and rejection rule to test the null hypothesis at the 1% significance level, using a sample of 20 technology stock P/E ratios.

(c) Suppose the average P/E ratio of the twenty stocks chosen is 43, with $s = 15$. What would be the analyst’s conclusion after performing this hypothesis test?
(d) The P-value of the result in (c) lies in which of the following intervals? Circle one, and justify your answer.

(i) more than .10
(ii) .05 to .10
(iii) .02 to .05
(iv) .01 to .02
(v) .005 to .01
(vi) less than .005

9. One of the largest firms making and marketing popular music has changed its manager responsible for acquiring new musical talent. In the past, at least 35% of recording contracts have resulted in hits. The higher level executives wish to find out if this proportion has decreased with the new manager. They decide that they will collect data on how many of the recording contracts made by the new manager have resulted in hits.

(a) State the appropriate null and alternative hypotheses.

(b) Choose a test statistic and rejection rule to test the null hypothesis at the 5% significance level, using a sample of 70 new contracts.

(c) The sample of 70 contracts resulted in 17 hits. What can you conclude about the proportion of recording contracts resulting in hits? Clearly explain your reasoning.

(d) Determine the P-value of the result in (c).

(e) Without prejudice to your answer to part c), what would be a type I error conclusion?

10. **Problem 9 (10 pts).** A random variable $X$ has probability density function as given below:

![Probability Density Function](image)

25 independent random variables $X_1, X_2, X_3, \ldots, X_{25}$ with this density function are averaged, to yield

$$
\bar{X} = \frac{X_1 + X_2 + X_3 + \ldots + X_{25}}{25}.
$$

Which of the following graphs most closely resembles the probability density function for $\bar{X}$?
11. Consider the following:

This is an example of how things could be worded in a way that you have never seen before, and that is why this past final example is on the review.

(a) Let $W$ be a normally distributed random variable with $E(W)=15$ and $P(12<W<18)=0.74$. Find the standard deviation.

(b) Assume $X$ is normally distributed with mean 7 and variance 4. Find a value $c$ such that $P(X>c)=0.42$.

(c) $Y$ is a normally distributed random variable with mean 20 and standard deviation 6. Find $P(|Y-21|>6)$. 