Math 218 Supplemental Instruction  
Spring 2008 Final Review – Part A

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Chapters 3, 4, and 5 Topics Covered:

- General probability (probability laws, conditional, joint probabilities, independence)
- Probability trees and Bayes’ Theorem
- Contingency tables
- Discrete random variables (probability distribution, cumulative probability distribution, mean, variance, standard deviation, expected value and variance laws)
- Permutations and combinations
- Continuous random variables (pdf, cdf, mean and variance using the pdf)
- Probability distributions: binomial, hypergeometric, uniform, Poisson, exponential, Poisson-exponential
- Normal distribution, standard normal distribution, Z table, Z transformation

A Couple Pointers:

- You can have a handwritten cheat sheet: 2 sides of an 8.5 x 11 page
- Round everything (work and answers) to at least 4 decimal places!!
- Remember to show one example of how to simplify a combination by hand
- Show all work: if you plug everything into your calculator and spit out an answer, you will not get any credit
- Make a list of all the distribution we’ve covered and match each question to the right distribution
- In Poisson-Exponential, first figure out which distribution applies to each part (time or distance interval = exponential, # of occurrences = Poisson)
- Make sure your calculator has batteries (and bring extra)!

Good luck!!!
1. A carnival has 3 games. In Game A, there is a 4% chance of winning, in Game B, the player has a 3% chance, and in Game C, a 2% chance. It is equally likely that you'll play any of the three games.
   
   a. Draw a tree diagram for this situation. Include all events and probabilities.
   
   b. Find the probability that you play Game A and win.
   
   c. Find the probability of winning.
   
   d. You arrive at the carnival to see that your friend has won a stuffed animal. What is the probability that she won it at Game A? (Use Bayes’ Theorem)

2. The average number of home games attended by USC students is represented in the following probability table. Y = the number of games a student attended:

<table>
<thead>
<tr>
<th>Y</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(Y)</td>
<td>0.07</td>
<td>0.13</td>
<td>0.14</td>
<td>0.24</td>
<td>0.25</td>
<td>0.11</td>
<td>0.06</td>
</tr>
</tbody>
</table>

   a. Find the probability that a random student attended more than 3 games.
   
   b. Calculate the expected value, variance, and standard deviation for Y.
   * Formula for E(Y):  
   * Formula for V(Y):
3. Your professor comes to class at most three minutes before it starts but is never late. Let the following function represent the probability of his early arrival:

\[ f(y) = \frac{4}{3}(y - \frac{2}{3}y^2 + \frac{1}{9}y^3) \quad 0 < y < 3 \]

a. Find the probability that he arrives at most two minutes and forty seconds before class starts.

b. Find the mean of this distribution. *Formula for E(Y):

c. Find the variance of this distribution. *Formula for V(Y):

4. The joint distribution of random variables X and Y is given below:

<table>
<thead>
<tr>
<th>X</th>
<th>Y 0</th>
<th>Y 1</th>
<th>Y 2</th>
<th>marginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.36</td>
<td>0.18</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.18</td>
<td>0.09</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.06</td>
<td>0.03</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Given:
\[ E(X) = 4.5 \]
\[ E(Y) = 0.5 \]

a. Find P(X > 5, Y > 0)

b. Prove that X and Y are independent.

c. Calculate the covariance of X and Y.

d. Calculate the correlation of random variables X and Y.
5. The amount of time it takes students at a local elementary school to finish their homework is uniformly distributed between 0.5 and 3 hours.

   a. Draw and label the graph for this distribution.

   b. What is the probability that a student will finish within 1.5 hours, given he’ll need more than an hour?

   c. Find the expected value and variance of the time needed for assignments.

   d. The principal wants to help the students who take the longest 15% to finish their assignments. What completion time should be used as the cutoff point?

   e. Find the probability that on exactly 2 days out of the 5 day week, a student completes his homework is less than an hour.

6. Find the following probabilities using the Z table:

   a. \( P (0 < Z < 2.53) \)  
   b. \( P (Z > 2.53) \)

   c. \( P(-1.23 < Z < 0) \)  
   d. \( P(-1.25 < Z < 0.12) \)

   e. \( Z_{0.01} \)
7. Scores on the Graduate Record Examination (GRE) are normally distributed with a mean score of 500 and a standard deviation of 75.
   a. Find the probability that a student’s GRE score will fall between 450 and 600.
   b. Find the probability that a student’s score will exceed 550.
   c. What score is needed so that only 5% of all scores will be higher than yours?

8. Since the Nintendo Wii is in such high demand right now, the likelihood of finding it in stock at any retail store is only 10%. Suppose we select 15 stores at random.
   a. Find the probability that the Wii is in stock at exactly 3 out of the 15 stores.
   b. What is the probability that at least one store has the Wii in stock?
   c. Find the expected value and standard deviation of the number of stores that will have the system in stock.

9. The average number of students entering the bookstore is Poisson distributed with an average rate of 3 per minute.
   a. What is the probability that at most 1 student enters the bookstore in the next 2 minutes?
   b. Find the probability that the next student arrives within half a minute.
   c. What is the probability that no students enter within the next two minutes.
   d. What is the average time between customers?
10. A travel agency is made up of 10 bilingual agents and 6 multilingual agents. A team of 4 will be chosen to put together a new series of tour packages.

a. What is the probability that at least 3 agents on the team will be multilingual?

b. Find the expected value and variance of the number of bilingual agents on the team.

c. If a new team of 4 is formed every year, what is the probability that in 7 out of the 10 years, there are at least 3 multilingual agents on the team?

11. A die is rolled. Event A = even number. Event B = number less than 3. Event C = number greater than 3.

a. Find P(A), P(B), and P(C).

b. Find P(B or C).

c. Find P(A or C).

d. Find P(A and B).

e. Find P(C | A).

f. Are events A and B independent?
Extra Practice Problems

1. The average number of passing grades given in Marshall’s toughest upper division courses is illustrated in the table below, where Y = the number of passing grades.

<table>
<thead>
<tr>
<th>Y</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(Y)</td>
<td>0.51</td>
<td>0.03</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.12</td>
<td>0.28</td>
</tr>
</tbody>
</table>

a. What is the probability that at least seven randomly chosen students within one of these classes passed their particular class?
b. Calculate the expected value and standard deviation of Y.

2. During a 1 hour TV show, the probability that you lose interest in the show is given by the following function, where Y is the time in hours that your interest lasted:

\[ f(y) = 10(2y^3 - 2y^4) \quad 0 < y < 1 \]

a. Find the probability that it takes you at most 35 minutes to lose interest.
b. Calculate \( P(0.2 < Y < 0.7) \)
c. Find the mean, variance, and standard deviation of the continuous function.

3. The joint distribution of random variables X and Y is given below:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>marginal</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
<td>0.05</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.05</td>
<td>0.05</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>marginal</th>
<th>1</th>
</tr>
</thead>
</table>

a. Find \( P(X > 1, Y < 1) \).
b. Calculate the covariance.
c. Calculate the correlation of X and Y.

4. Use the Z table to find the following.

a. \( P(Z < 1.26) \)
b. \( P(0.17 < Z < 2.31) \)
c. \( P(Z < -2.11) \)
d. \( P(-1.21 < Z < 0.42) \)
e. Find \( Z_{0.01} \)
f. Find \( Z_{0.025} \)
5. A random sample of 50 purchases at a department store produced the following contingency table for the method of payment and the size of the purchase:

<table>
<thead>
<tr>
<th></th>
<th>Cash</th>
<th>Credit Card</th>
<th>Debit card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under $30</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$30 - $100</td>
<td>2</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Over $100</td>
<td>1</td>
<td>12</td>
<td>7</td>
</tr>
</tbody>
</table>

   a. Find the probability that a randomly selected purchase is either under $30 or was paid by debit card.
   b. Given that a purchase was made on debit, find the probability that it was at least $30.
   c. Find the probability that a purchase was paid by cash.
   d. Are “under $30” and “cash” independent events? Justify your answer.

6. Three airlines serve a small town in Alaska. Airline A has 40% of all scheduled flights, airline B has 25%, and airline C has the remaining 35%. Their on-time rates are 85%, 65%, and 70%, respectively.
   a. Draw a tree diagram and insert the probabilities for each branch.
   b. Find the probability that a flight is delayed, given that it’s airline B’s flight.
   c. Find the probability that a random flight is airline B’s and is on-time.
   d. Find the probability that a randomly selected flight is on-time.
   e. A plane has just left on-time. What is the probability that it was airline A’s? (Use Bayes’ theorem)

7. 3 members out of a 10 member swim team will be sent to the upcoming meet.
   a. If the 3 members are ranked, how many different ways can they be selected?
   b. If, instead, they go as a team, and ranking isn’t an issue, how many ways can the team be selected from the 10 members?

8. The number of people at an ATM is Poisson distributed with a mean of 25 per hour.
   a. Find the probability that at least 3 people go to the ATM in 20 minutes.
   b. What is the probability that 6 minutes elapses before the next person arrives?
   c. What is the average interval of time between consecutive people?

9. Five cards are randomly drawn from a deck of cards.
   a. Find the probability that you draw exactly 3 diamonds.
   b. Find the probability that you draw 3 or more diamonds.
   c. How many diamonds do you expect to find when you draw 5 cards?

10. Ten percent of milk sold at a supermarket is past its expiration date. Twenty milk cartons are selected at random.
    a. Find the probability that five cartons will be past their expiration date.
    b. Find the probability that at least one will be past its expiration date.
    c. What is the mean and standard deviation of the number of milk cartons that are past their expiration date?
Extra Practice Problem Answers

1. a. 0.41  
   b. 5.47; 2.7330

2. a. 0.3088  
   b. 0.5215  
   c. 0.6667; 0.0317; 0.1781

3. a. 0.6  
   b. -0.1  
   c. -0.2222

4. a. 0.8962  
   b. 0.4221  
   c. 0.0174  
   d. 0.5497  
   e. 2.33  
   f. 1.96

5. a. 0.5  
   b. 0.8889  
   c. 0.16  
   d. not independent

6. a. (tree diagram)  
   b. 0.35  
   c. 0.1625  
   d. 0.7475  
   e. 0.4548

7. a. 702  
   b. 120

8. a. 0.9895  
   b. 0.0821  
   c. 2.4 minutes

9. a. 0.0815  
   b. 0.0927  
   c. 1.25

10. a. 0.0319  
    b. 0.8784  
    c. 2; 1.3416